

# Skyrmions: From 3D chiral magnets to 2D oxide interfaces

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Indo-US Bilateral Workshop on  
Physics and Chemistry of Oxides: Theory meets Experiment  
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1. SB, J. Rowland, O. Erten & M. Randeria, Phys. Rev. X **4**, 031045 (2014).
2. J. Rowland, SB & M. Randeria, Phys. Rev. B (Rapid) **93**, 020404 (2016).
3. SB, O. Erten & M. Randeria, Nature Physics **9**, 626 (2013).

# Chiral magnetism

Chiral Dzyaloshinskii-Moriya (DM) exchange

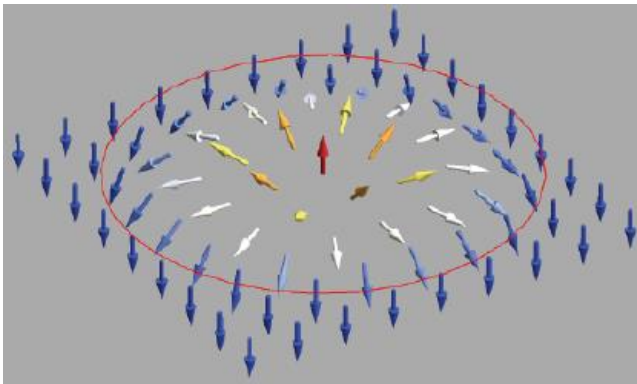
$$D_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$$

$$D\mathbf{m} \cdot (\nabla \times \mathbf{m})$$

Broken inversion, e.g.  $(x, y, z) \not\rightarrow (-x, -y, -z)$

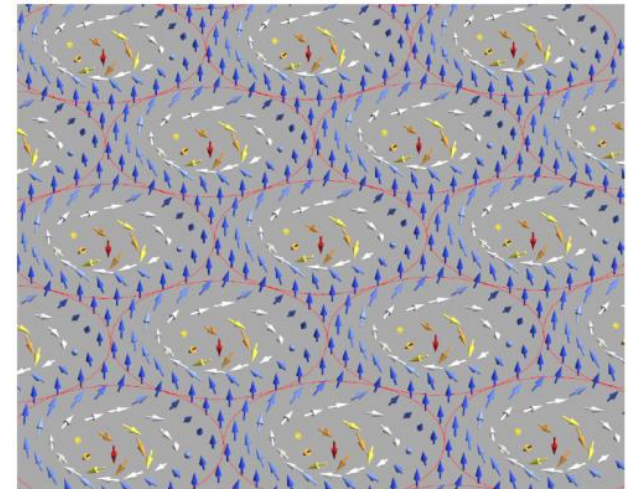
→ “Chiral magnets”

Skyrmions



Topological spin texture

Skyrmion crystal (SkX)



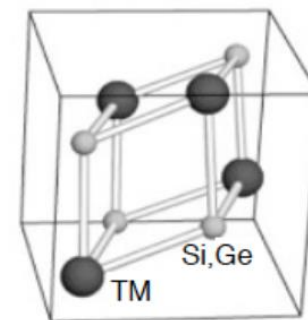
# Conventional 3D chiral magnets

- Non-centrosymmetric crystals

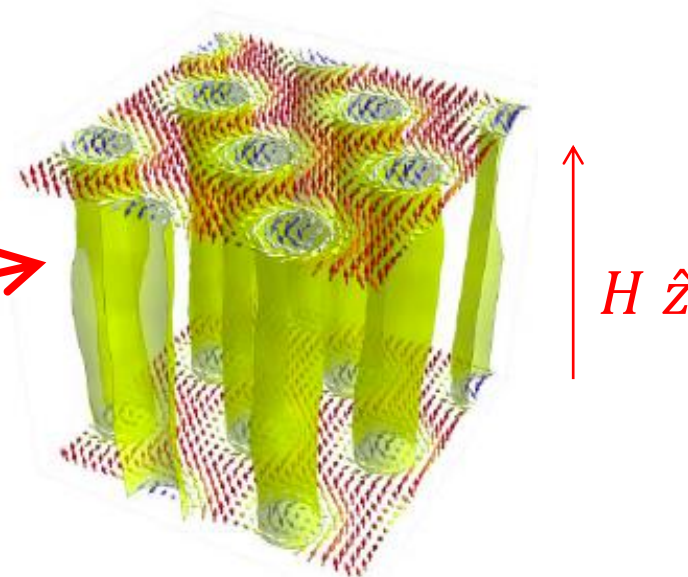
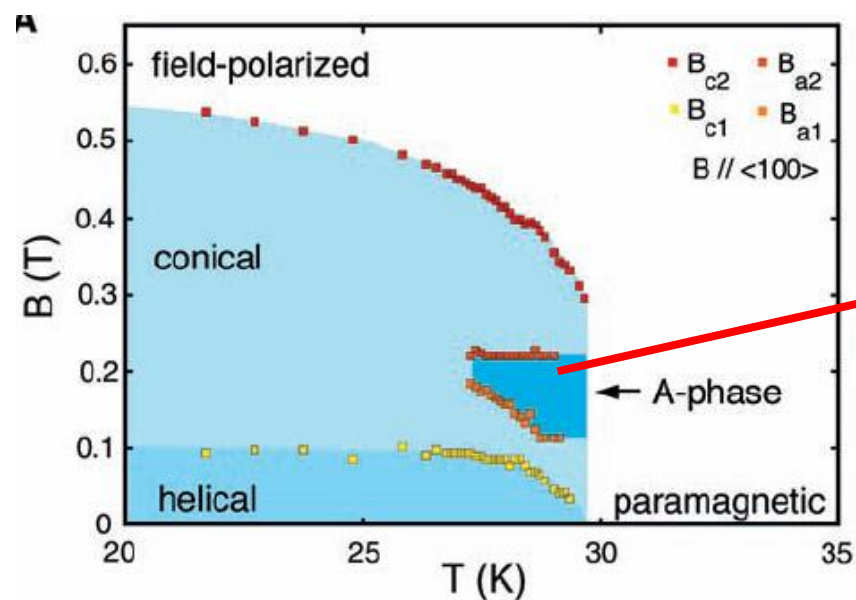
Cubic crystals without bulk inversion symmetry

Metals: **MnSi**,  $\text{Fe}_{1-x}\text{Co}_x\text{Si}$ , FeGe, ...

Insulators:  $\text{Cu}_2\text{OSeO}_3$



B20: no inversion center



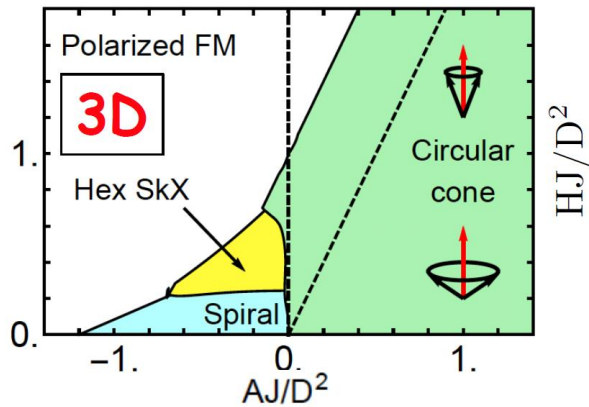
**Skyrmion crystal** – Narrow region in the phase diagram at finite temperature

# How to enhance the stability of skyrmion phases?

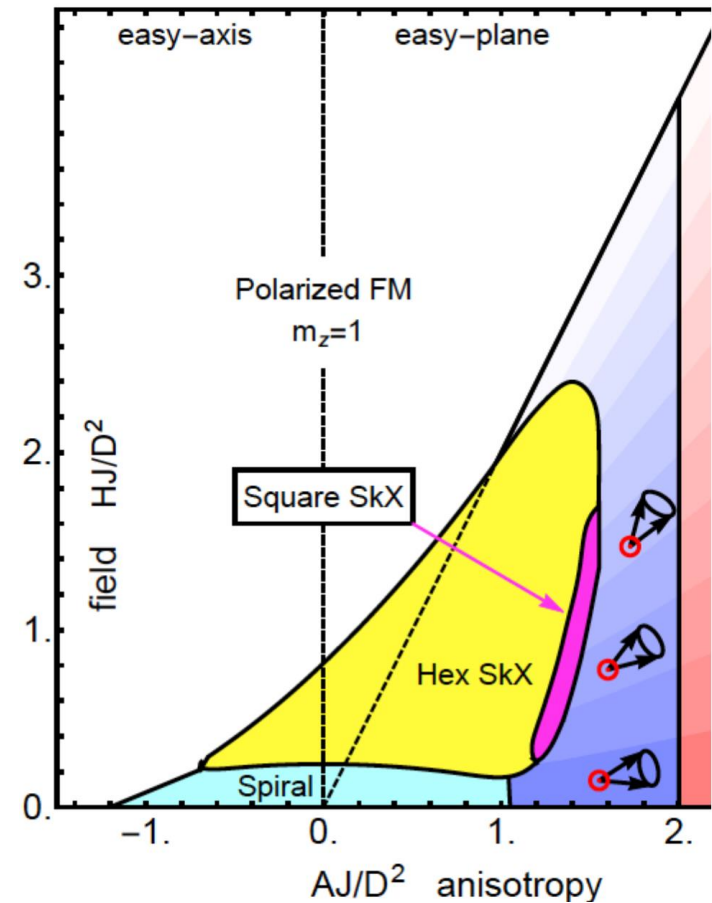
e.g. SkX ground state.

By tuning materials, dimensionality, ...

→ Phenomenological and microscopic understanding of the mechanism for stability

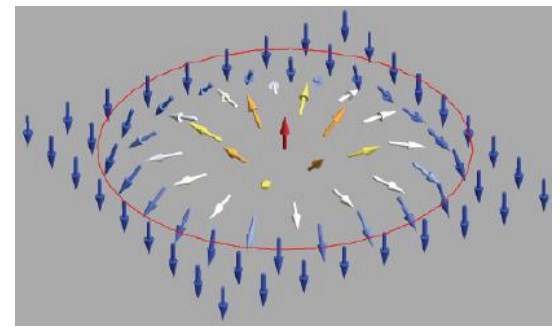


Dimensionality  
and/or  
Symmetry



→ “Uncover” much richer phase diagram  
e.g. Transition between SkX phases

## Broader motivation



Magnetic Skyrmions: Topological spin texture

- {Condensed matter physics}  $\cap$  {Topology}

Topological phenomena – “Emergent electromagnetism”

→ Topological Hall effect, spin transfer torque,  
skyrmion-flow Hall effect

- Interplay of magnetism and spin-orbit coupling (SOC)

→ New materials, Novel magnetic phases

- Long-wavelength  
structures

in

Correlated quantum materials



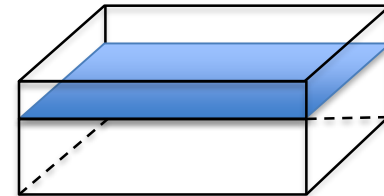
“Soft” condensed matter



“Hard” condensed matter

# Outline

- Skyrmions and chiral magnetism.
- Skyrmion materials and properties.
- How to stabilize skyrmion phases?  
System with broken mirror symmetry ( $z \not\rightarrow -z$ )  
→ Much richer skyrmion phase diagram
- Conclusions and outlook.



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System with broken mirror symmetry ( $z \rightarrow -z$ )  
→ Much richer skyrmion phase diagram

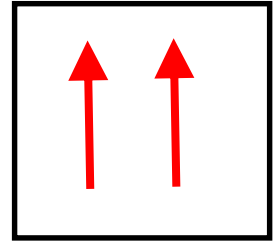
- Conclusions and outlook.



# Chiral magnetism : DM term

- Magnetism → magnetic exchange (isotropic)  
(Local moments)

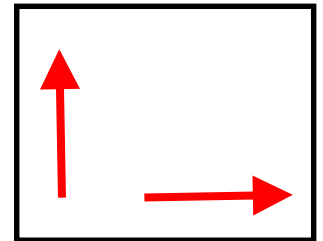
$$\mathcal{H}_J = -J \mathbf{S}_i \cdot \mathbf{S}_j$$



- Broken inversion e.g.  $(x, y, z) \not\rightarrow (-x, -y, -z)$

→ Chiral Dzyaloshinskii-Moriya exchange

$$\mathcal{H}_{DM} = \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$$



Symmetry → Direction of  $\mathbf{D}$  vectors

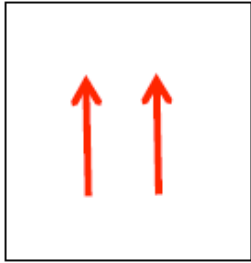
Microscopics → Magnitude of  $\mathbf{D}$  vectors

$J \gg D \propto \alpha \leftarrow$  Spin-orbit coupling strength

$$J > 0$$

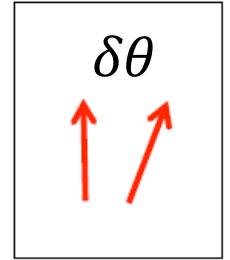
## Spin textures

## Twist



$$-J \mathbf{S}_i \cdot \mathbf{S}_j \sim -J \cos \delta\theta \sim J\delta\theta^2/2$$

$$-D \cdot (\mathbf{S}_i \times \mathbf{S}_j) \sim -D \sin \delta\theta \sim -D\delta\theta$$

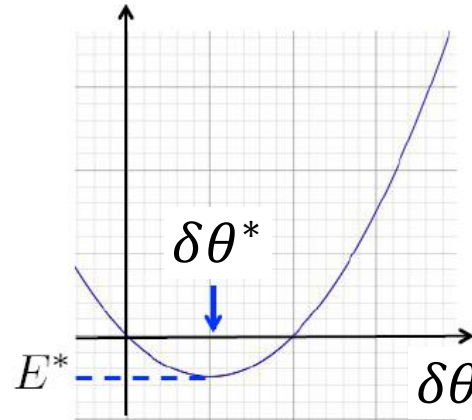


Pitch

$$\delta\theta^* = D/J$$

→ Length scale

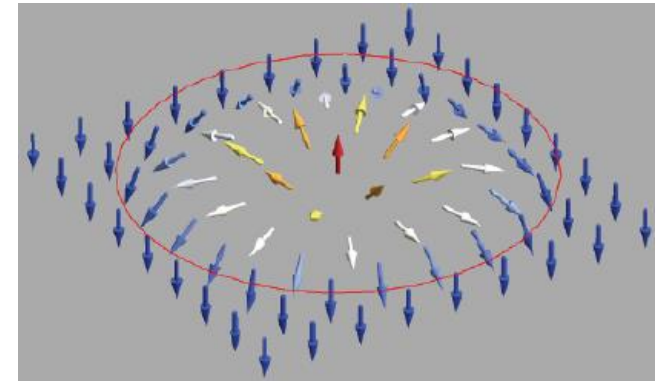
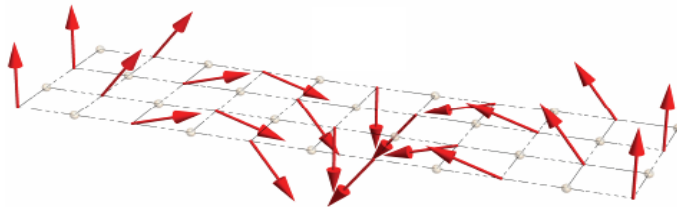
$$L_D = \left(\frac{J}{D}\right) a \gg a$$

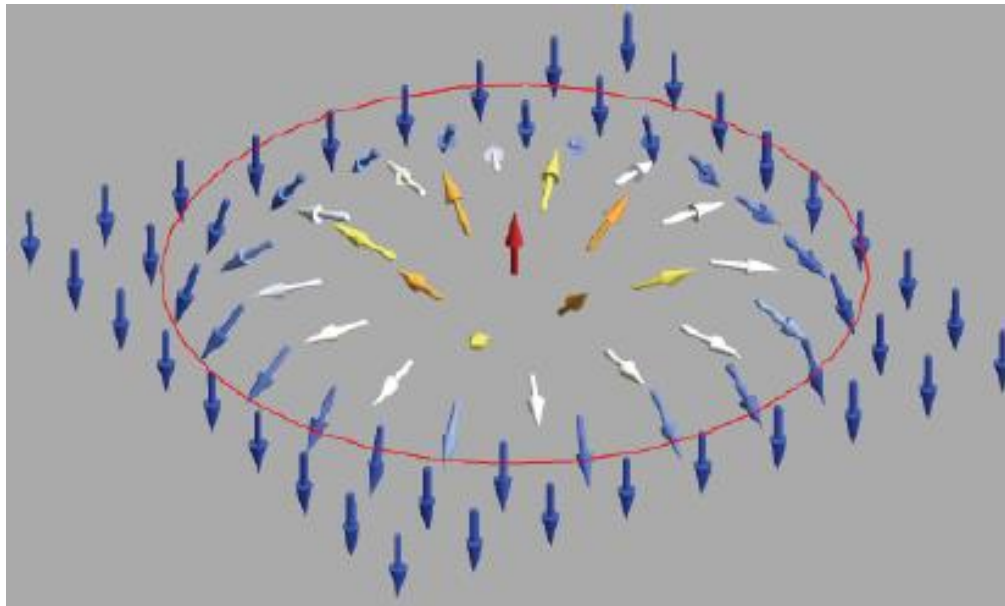


$$E^* = -D^2/2J$$

## Skyrmions

Spiral

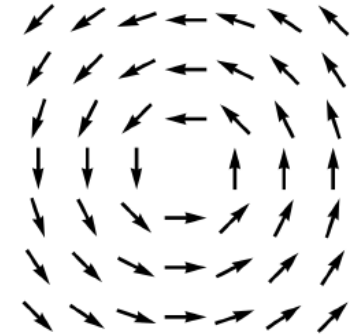




What is skyrmion?

**Skyrmion** → A topological soliton

Example: **Vortex**  
in superfluid



The first use of a **topological soliton**  
to model a particle

"A unified field theory of mesons and  
baryons"

**Nuclear Physics 31, 556 (1962)**

T. H. R. Skyrme  
(1922 – 1987)



**Skymions in condensed matter:**

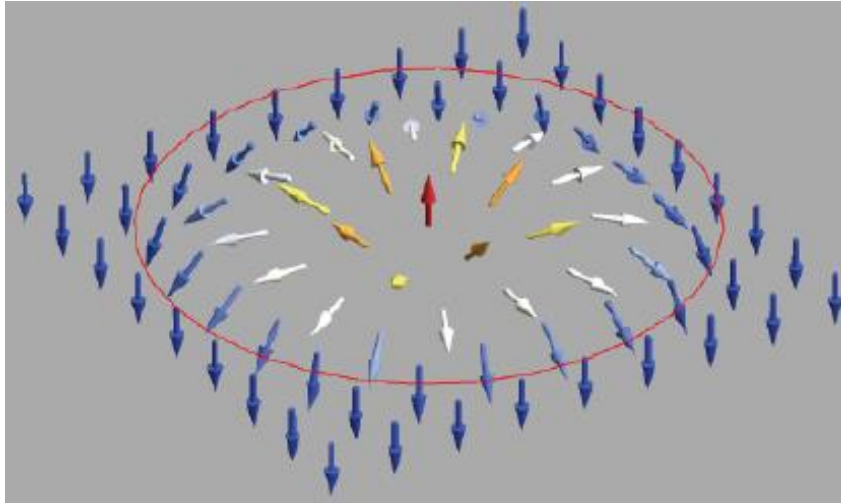
- Liquid crystals
- Quantum Hall effect
- Spinor Bose condensates
- Cold atoms with synthetic gauge fields
- Magnetic materials\*

→ **Chiral magnets**

\* Belavin-Polyakov solution for metastable state in 2D Heisenberg model

# Magnetic Skyrmions

Texture in local magnetization  $\hat{\mathbf{m}}(\mathbf{r})$



(Hedgehog or Ne'el skyrmion)

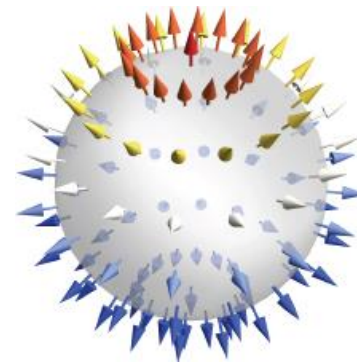
Topological charge

$$N_{sk} = \int d^2r \chi(\mathbf{r}) \\ = 0, \pm 1, \pm 2, \dots$$

Topological charge density

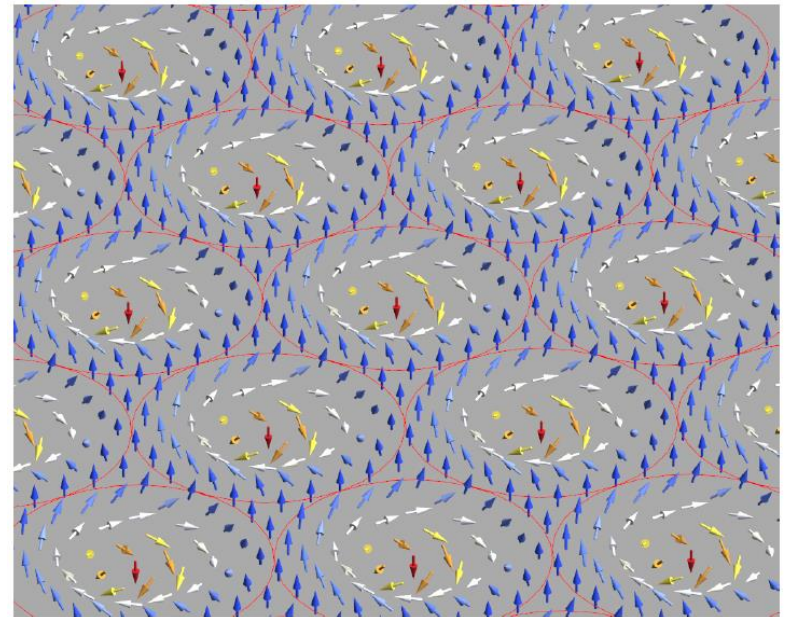
$$\chi(\mathbf{r}) = \frac{1}{4\pi} \hat{\mathbf{m}} \cdot (\partial_x \hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}})$$

“Hedgehog”



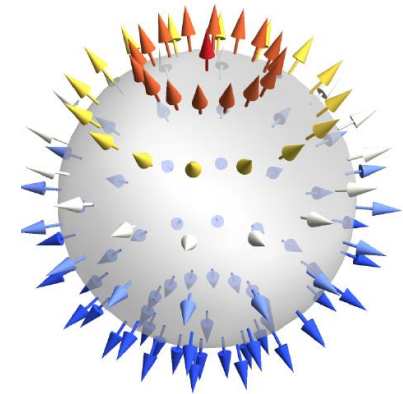
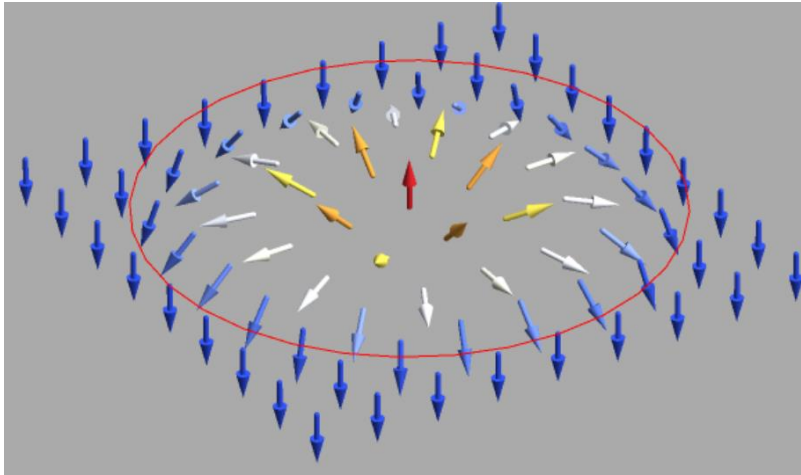
Wrapping of  
unit sphere  
in spin space

$$\Pi_2(S^2) = \mathbb{Z}$$

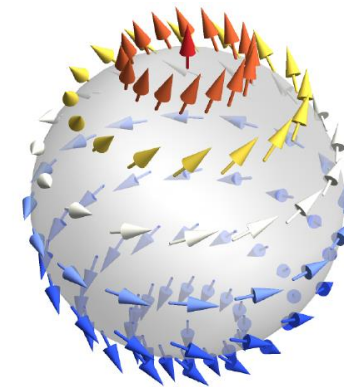
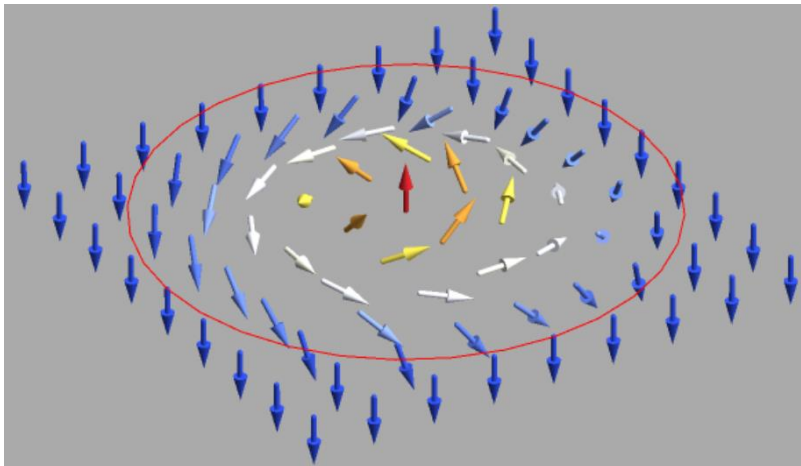


Skyrmion crystal (SkX)<sub>13</sub>

# Different types of skyrmions



Hedgehog or  
Ne'el skyrmion



Vortex or Bloch  
skyrmion

# Outline

- Skyrmions and chiral magnetism.

- **Skyrmion materials and properties.**

- How to stabilize skyrmion phases?

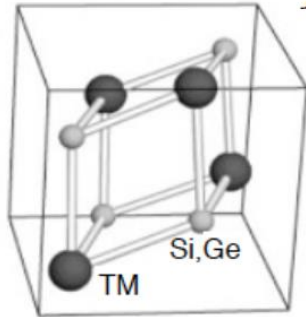
System with broken mirror symmetry ( $z \rightarrow -z$ )  
→ Much richer skyrmion phase diagram

- Conclusions and outlook.

# Skyrmions in 3D chiral magnets

- Non-centrosymmetric crystals

Cubic crystals without bulk inversion symmetry

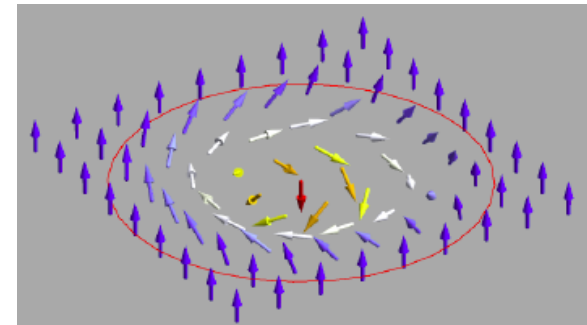


B20: no inversion center

$$(x, y, z) \not\rightarrow (-x, -y, -z)$$

DM interaction

$$D\mathbf{m} \cdot (\nabla \times \mathbf{m})$$



Vortex or Bloch skyrmion

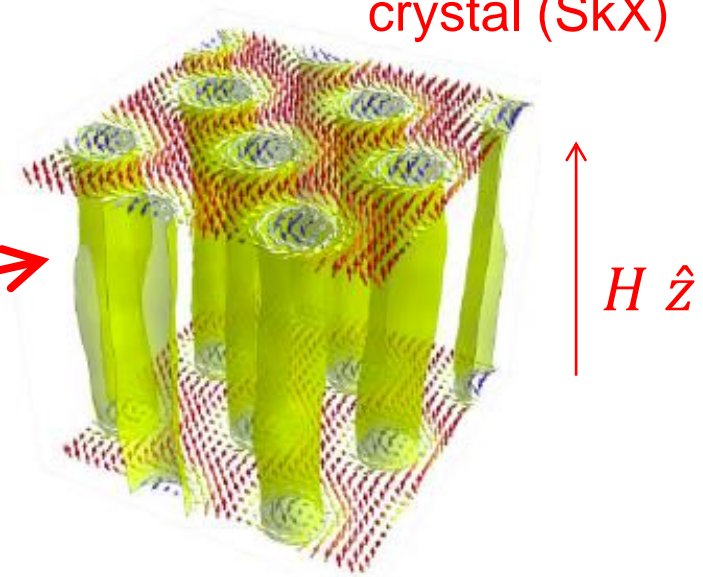
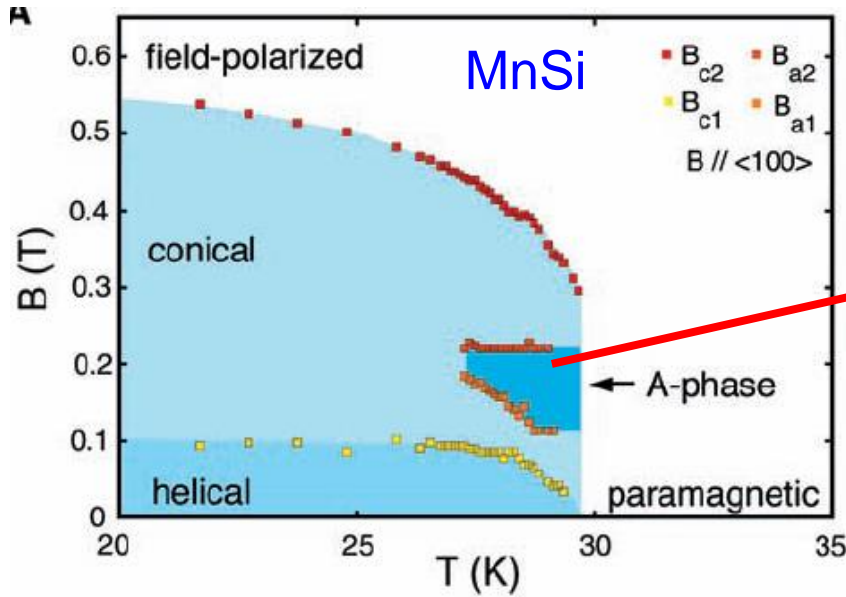
Experiments: C. Pfleiderer, Y. Tokura, ..

Theory: A. Bogdanov, N. Nagaosa, A. Rosch, A. Vishwanath, C. Batista ...<sup>16</sup>

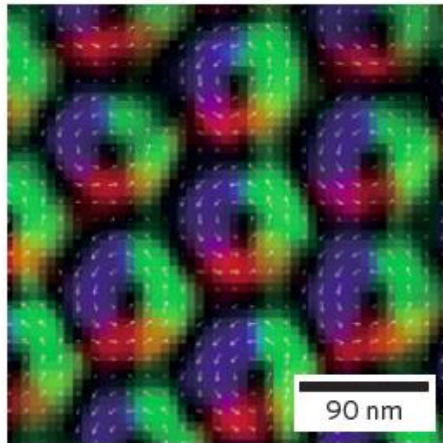


# Observation of skyrmions

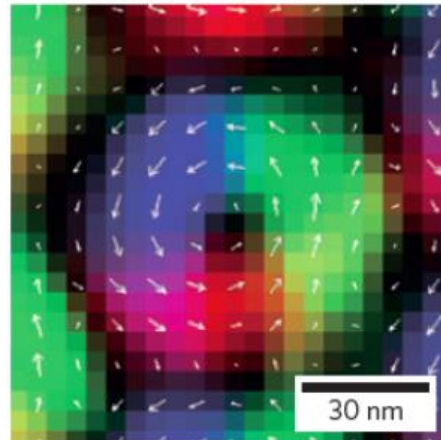
Skyrmion crystal (SkX)



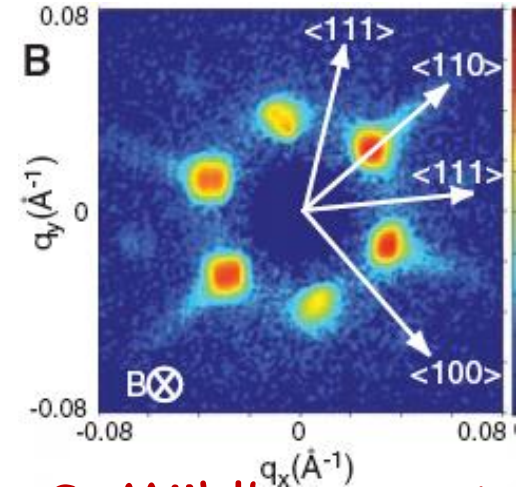
# Real-space observation of skyrmion



**FeCoSi**



# Neutron Scattering



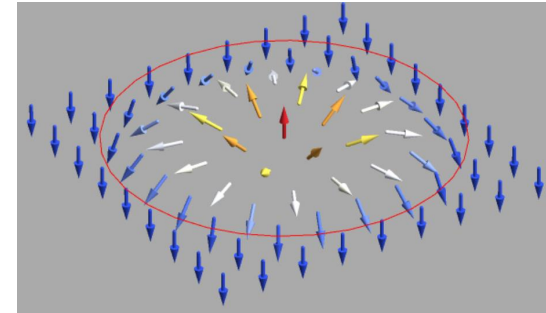
Lorentz TEM  
Tokura group

Yu et al., Nature (2010)

S. Mühlbauer, et al.  
Science (2009).<sup>17</sup>

# Chiral magnetic materials with broken surface inversion/mirror

$$z \xrightarrow{\text{X}} -z$$

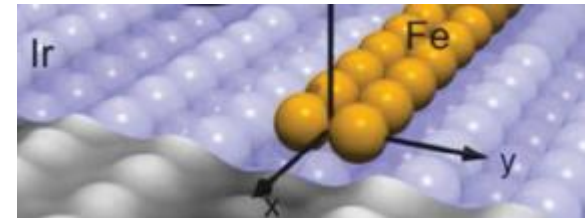


Hedgehog or Ne'el skyrmion

DM interaction

$$Dm \cdot ((\hat{z} \times \nabla) \times m)$$

- Magnetic monolayer on metals with large spin-orbit coupling.



Bode et al, Nature (2007); Romming et al., Science (2013))

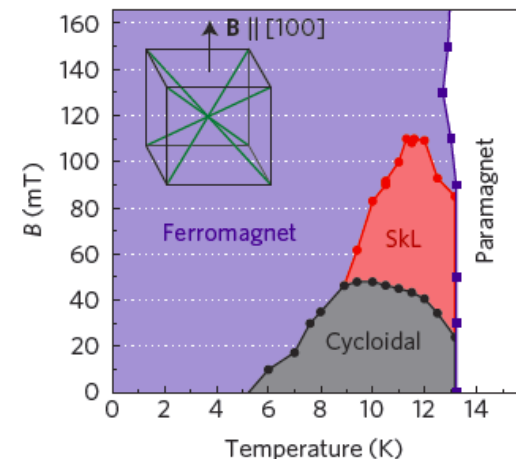
- Multilayers with interfacial DM interactions, room temperature skyrmion.

Moreau-Luchaire et al., Nature Nano (2016)

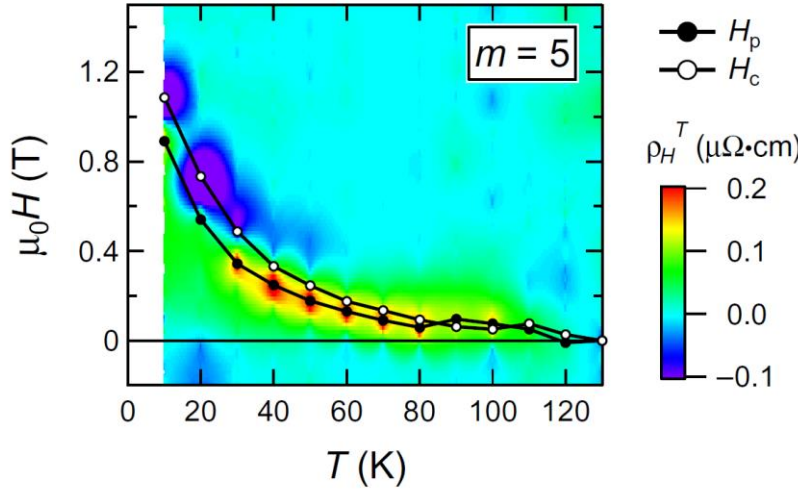
- New 3D skyrmion material with broken mirror

Polar magnetic semiconductor  $\text{GaV}_4\text{S}_8$

Kézmárki et al, Nature Mat. (2015)



○ Oxide interfaces → broken surface inversion and large Rashba SOC  
 $\text{SrRuO}_3/\text{SrIrO}_3$  Matsuno et al., Sci. Adv. (2016)



← Topological Hall effect

Potential candidates – Oxide Interfaces with magnetism

$\text{LaAlO}_3/\text{SrTiO}_3$  (?) – SB, Erten & Randeria, Nature Phys. (2013)  
 SB, Rowland, Erten & Randeria, PRX (2014)  
 Li, Liu & Balents., PRL (2014)

$\text{GdTiO}_3/\text{SrTiO}_3$  – Robust magnetism and large Rashba SOC  
 Expt: Moetakef et al., PRX (2012).  
 DFT: H. Banerjee, SB, M. Randeria & T. Saha-Dasgupta, Sci. Rep. (2015)

And may be many more .....

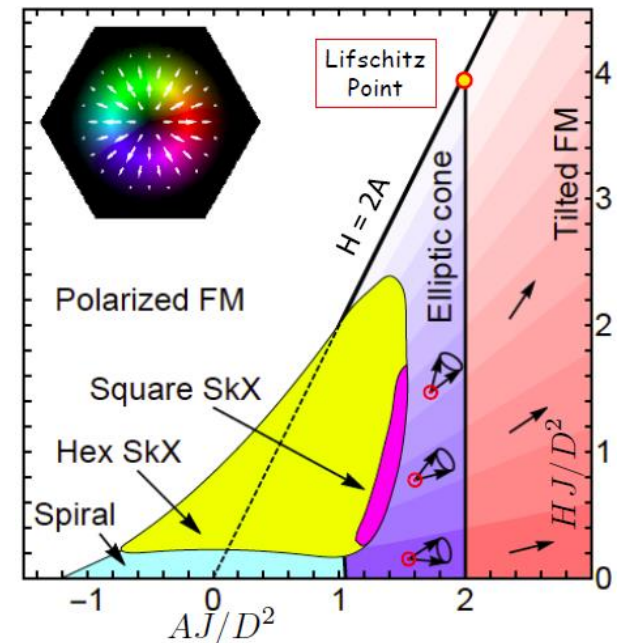
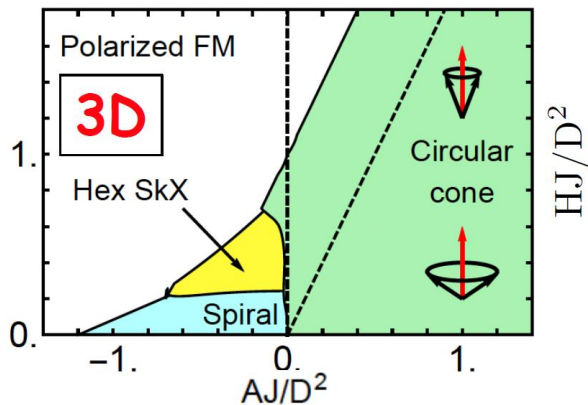
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→ Much richer skyrmion phase diagram

- Conclusions and outlook.



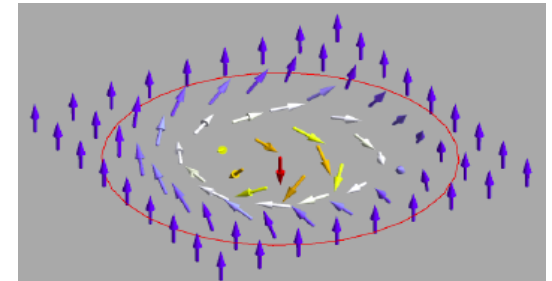
# Form of DM interaction $\mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$

Symmetry  $\rightarrow$  Direction of DM vector

SOC  $\rightarrow$  Magnitude of DM

- Broken bulk inversion  $\mathbf{r} \not\rightarrow -\mathbf{r}$   
“Dresselhaus SOC”

$$\rightarrow \hat{\mathbf{D}}_{ij}^D = \hat{\mathbf{r}}_{ij} \rightarrow D_D \mathbf{m} \cdot (\nabla \times \mathbf{m})$$



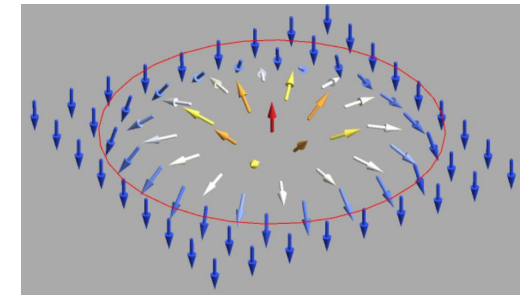
Bloch skyrmion

- Broken surface inversion/mirror

$$\mathbf{z} \not\rightarrow -\mathbf{z}$$

Rashba SOC

$$\rightarrow \hat{\mathbf{D}}_{ij}^R = \hat{\mathbf{z}} \times \hat{\mathbf{r}}_{ij} \rightarrow D_R \mathbf{m} \cdot ((\hat{\mathbf{z}} \times \nabla) \times \mathbf{m})$$



Ne'el skyrmion

Length scale,  $L_D = (J/D)a \gg a \rightarrow$  Continuum field theory

$T = 0$  constraint  $\mathbf{m}^2(\mathbf{r}) = 1$

$$\mathcal{F}[m(\mathbf{r})] = \frac{J}{2} |\nabla m|^2$$

+ DM terms

$$+ A m_z^2$$

$$- H m_z$$

$A > 0$  Easy plane  
 $A < 0$  Easy axis

Isotropic FM exchange

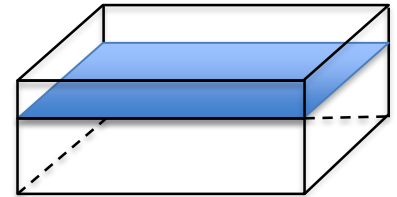
Dzyaloshinskii-Moriya

Anisotropy

Magnetic field

Anisotropy axis,  
Mirror plane  $\perp$

$H \hat{z}$



Origin of anisotropy:

$$A = A_s + A_c$$

$A_s$ , single-ion + dipolar

Rashba SOC  $\rightarrow A_c \simeq D^2/2J$ , Kitaev-compass anisotropy, always energetically comparable to DM

SB, Rowland, Erten & Randeria, PRX (2014)

→  $T=0$  Phase diagram as a function of

- Field  $HJ/D^2$
- Anisotropy  $AJ/D^2$

Minimize  $\mathcal{F}$  with the constraint  $m^2(\mathbf{r}) = 1$

→ Optimal  $m(\mathbf{r})$

- Variational calculations (analytical)
- Conjugate gradient minimization (numerical)

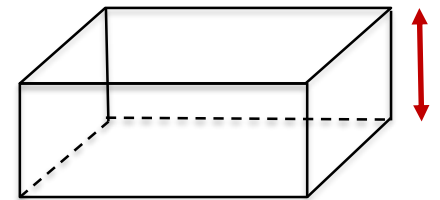
**3D systems** -- Bulk or film with thickness  $\gg (J/D)a$

→ Periodic boundary condition in  $x, y, z$

**2D systems** -- thickness  $\ll (J/D)a$

**3D systems with broken mirror symmetry**

→ no  $z$  variation



Broken bulk inversion and/or broken surface inversion/mirror

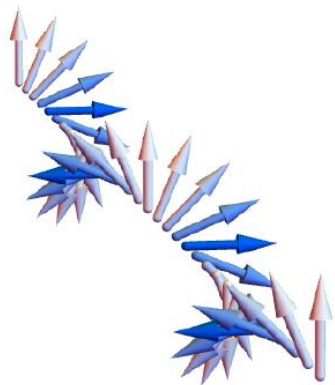
# 3D with broken bulk inversion

$T = 0$

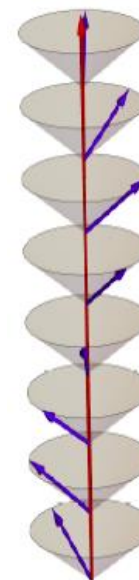
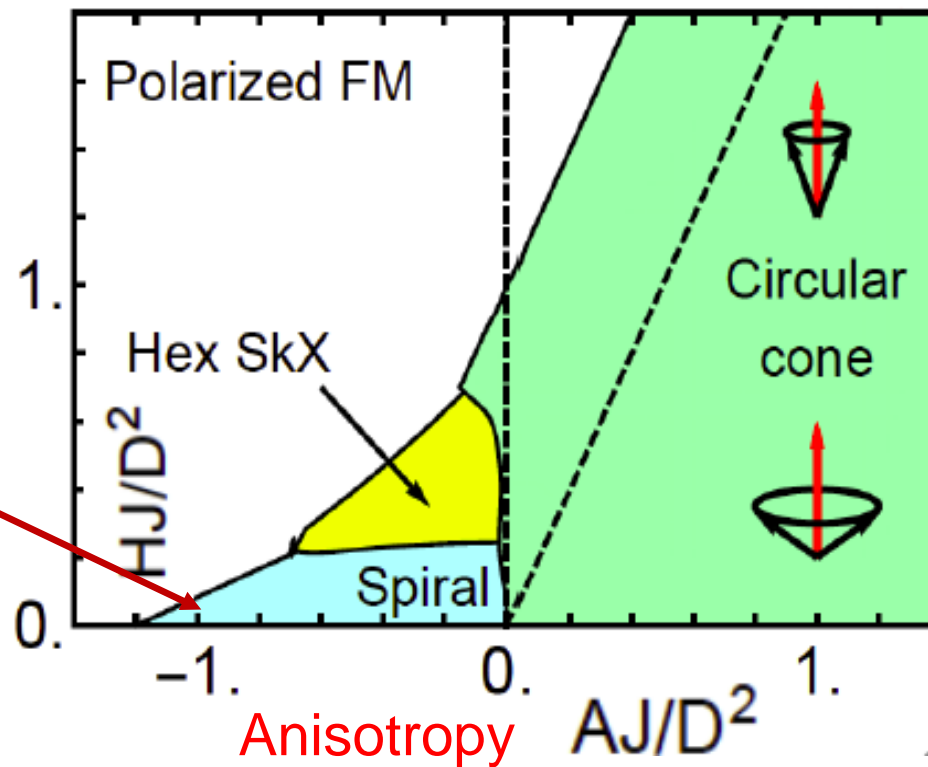
Easy axis

Easy plane

Helical or Bloch spiral

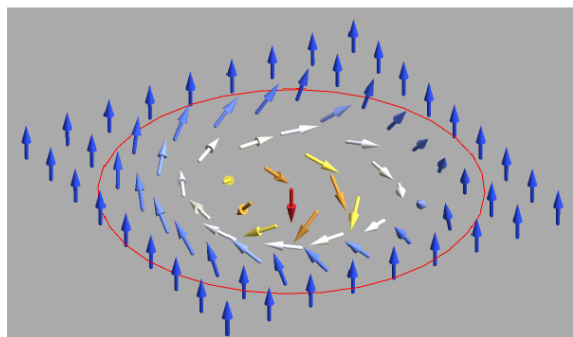


$HJ/D^2$  Field



Cone

Hexagonal crystal of Bloch skyrmion



Conventional chiral magnet e.g. MnSi

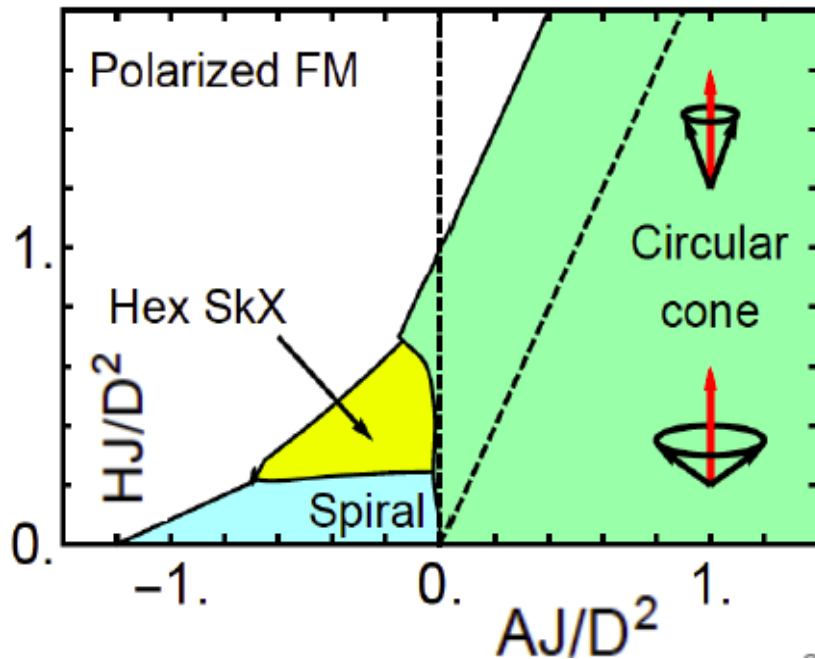
Wilson et al., PRB (2014)



## Conventional chiral magnets

Broken bulk inversion

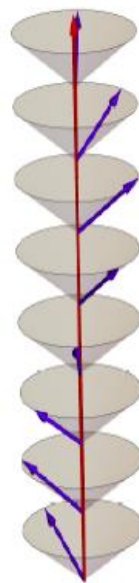
$$(x, y, z) \rightarrow (-x, -y, -z)$$



DM

$$D\mathbf{m} \cdot (\nabla \times \mathbf{m})$$

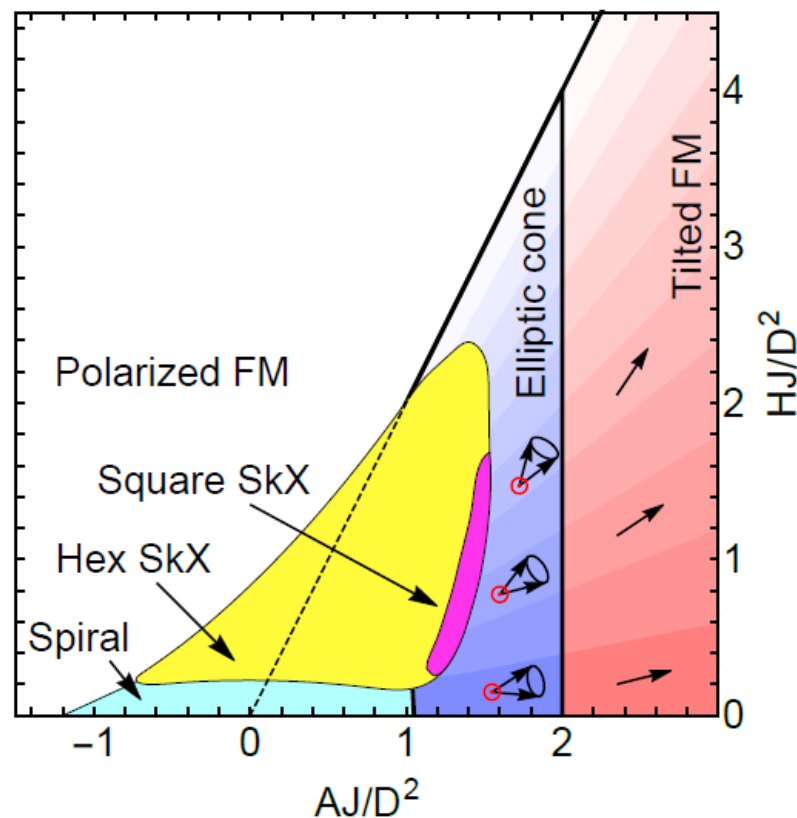
- Cone phase dominates.
- Twist in 3 directions.



Cone

## Chiral magnets with Rashba SOC

Broken  $z \rightarrow -z$

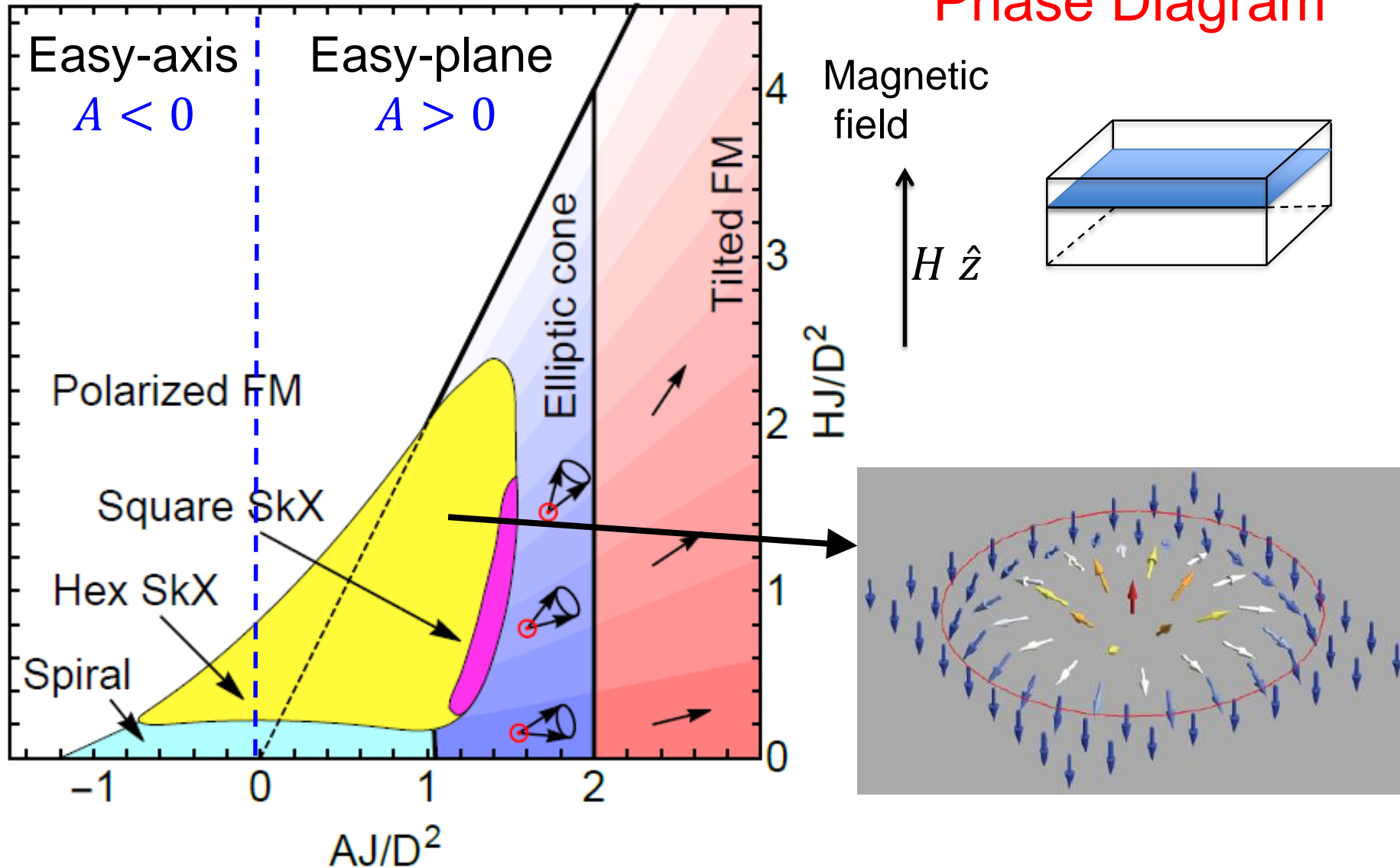


DM

$$D\mathbf{m} \cdot ((\hat{\mathbf{z}} \times \nabla) \times \mathbf{m})$$

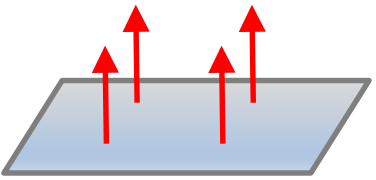
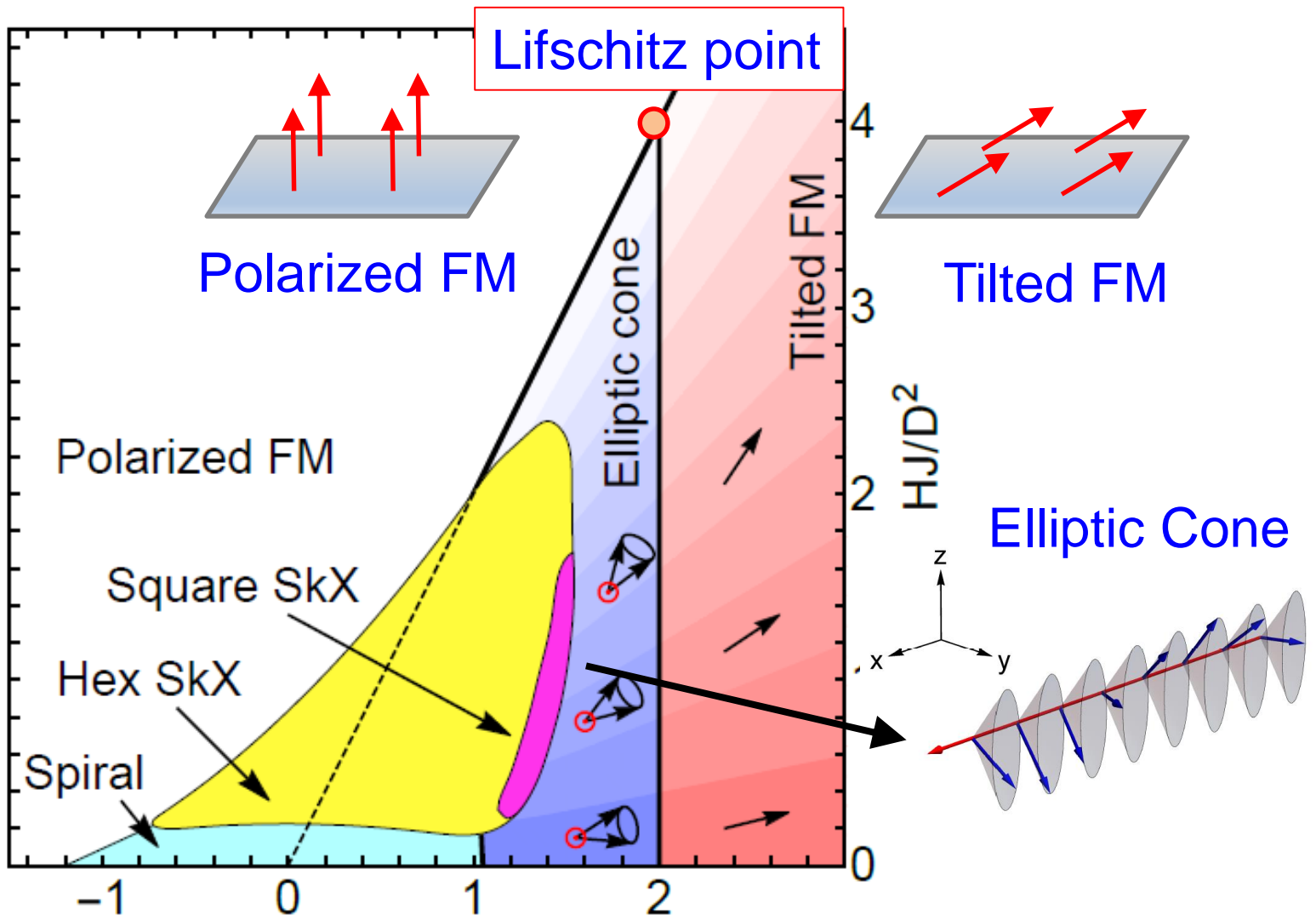
- No cone phase even in 3D with broken mirror.

# Phase Diagram

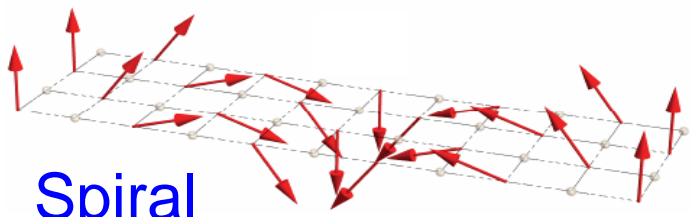


→ Very large region of stable skyrmion crystal (SkX) phase

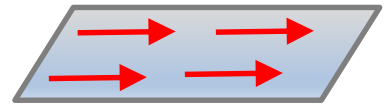
Rich  
Phase  
Diagram



out-of-plane FM



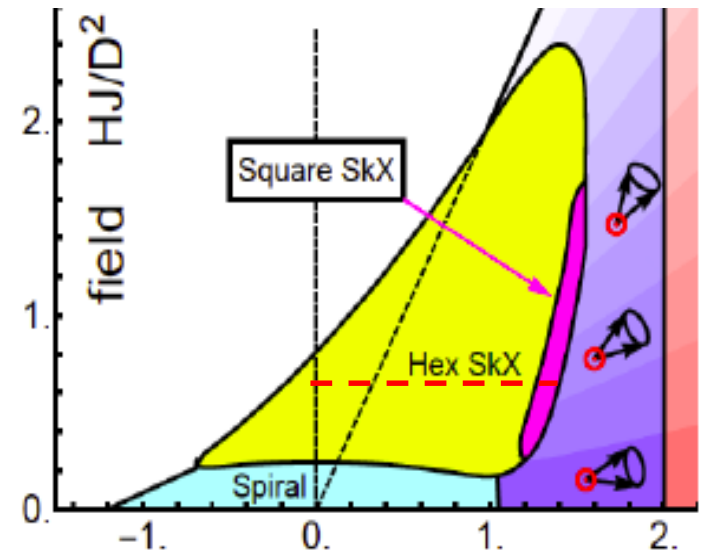
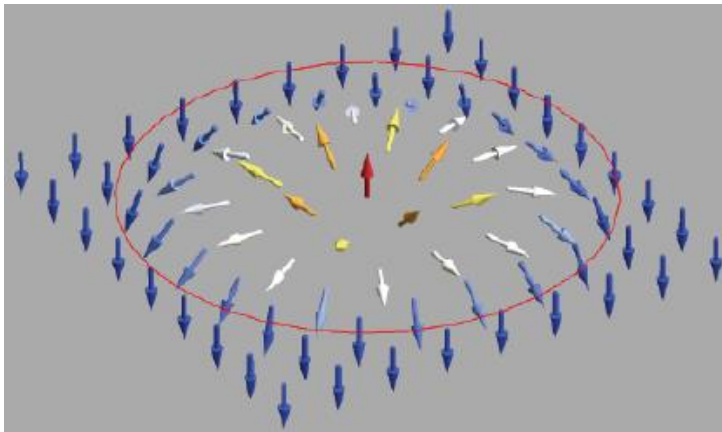
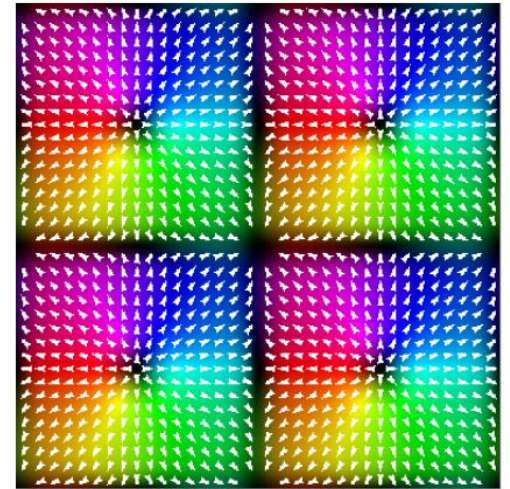
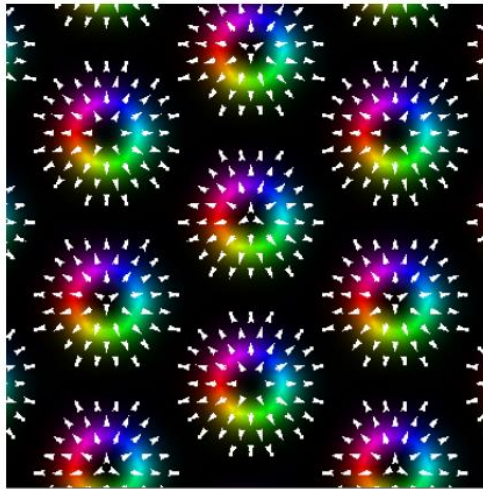
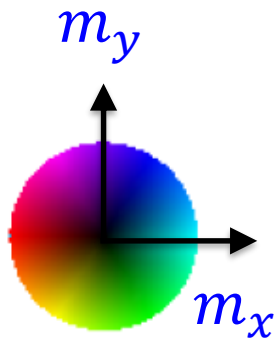
Spiral



in-plane FM

# Skyrmion crystal phases

Hexagonal SkX  $\rightarrow$  Square SkX transition  
tuned by **anisotropy**

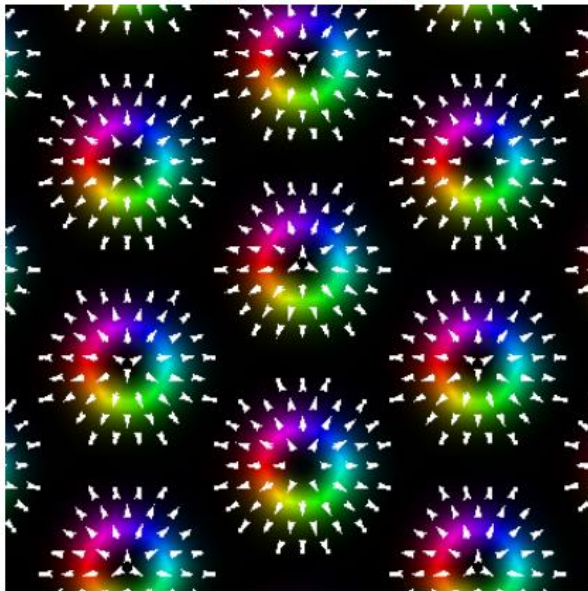


# “Conventional” Hexagonal Skymion crystal

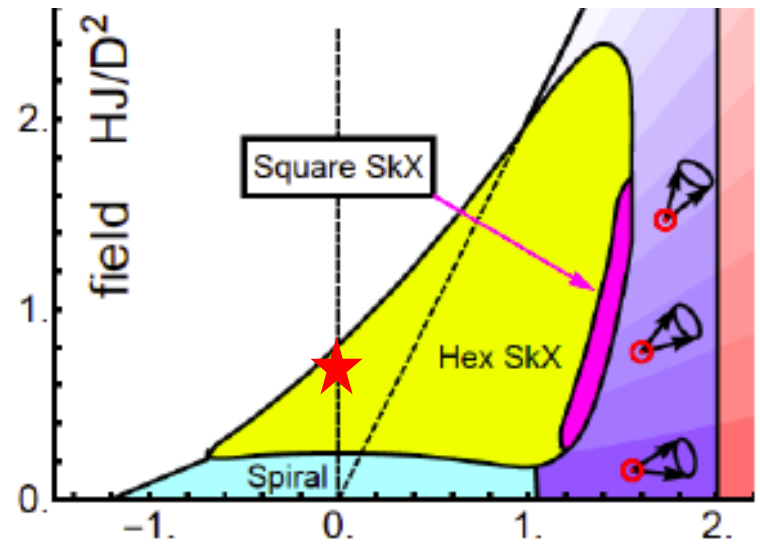
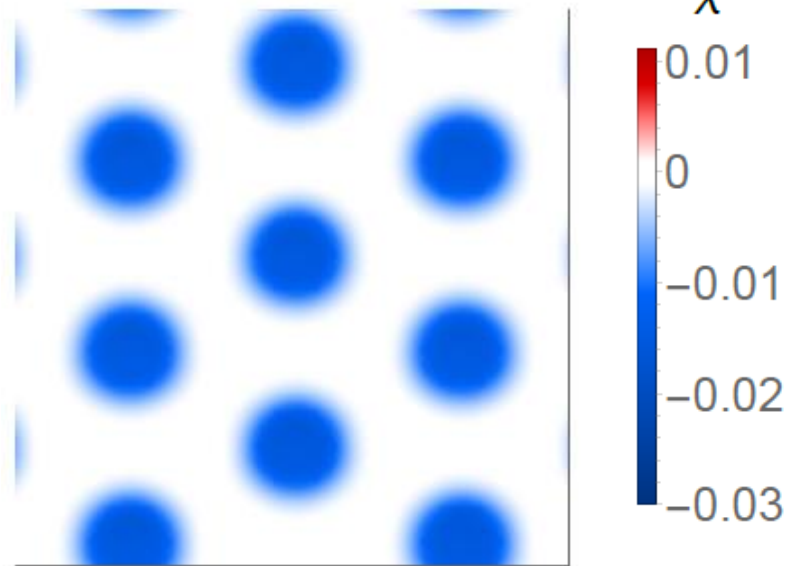
Topological charge density

$$\chi(\mathbf{r}) = \frac{1}{4\pi} \hat{\mathbf{m}} \cdot (\partial_x \hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}})$$

$AJ/D^2=0.0$



Topological charge  
Density  $\chi$



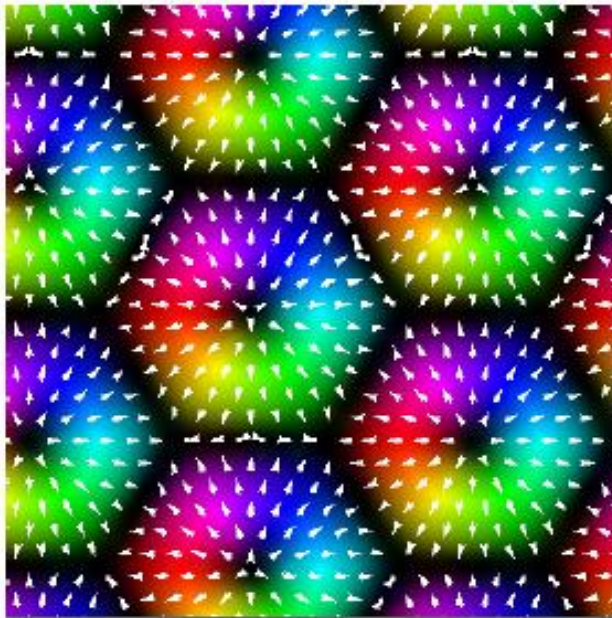
Topological charge,  $N_{sk} = \int_{u.c.} d^2r \chi(r) = -1$ <sub>29</sub>

# “Vortex-antivortex” Skyrmion crystal

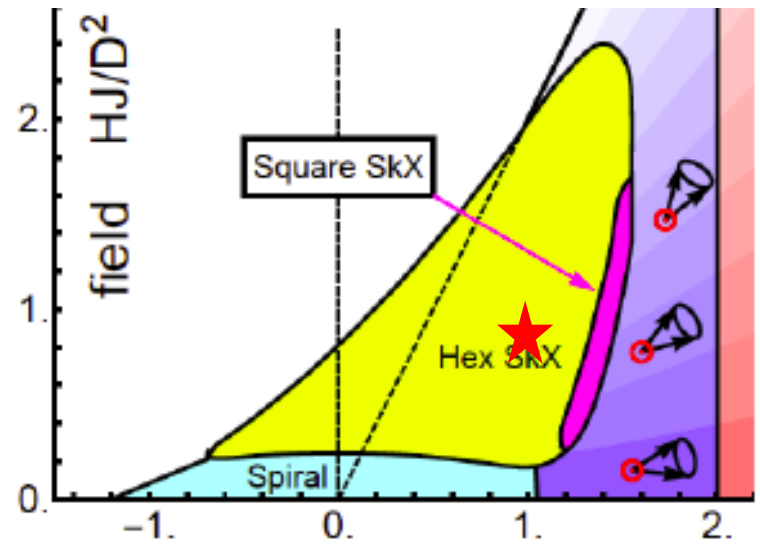
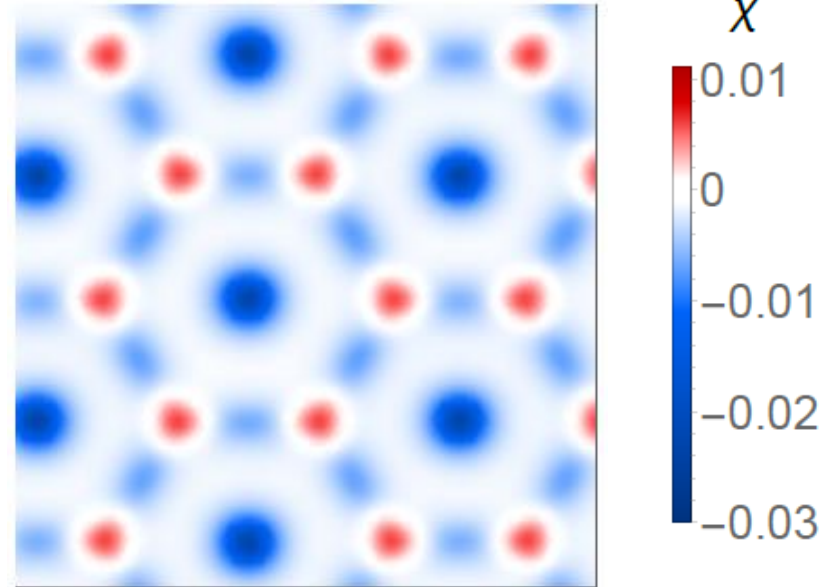
Topological charge density

$$\chi(\mathbf{r}) = \frac{1}{4\pi} \hat{\mathbf{m}} \cdot (\partial_x \hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}})$$

$$AJ/D^2 = 1.2$$



Topological charge  
Density  $\chi$



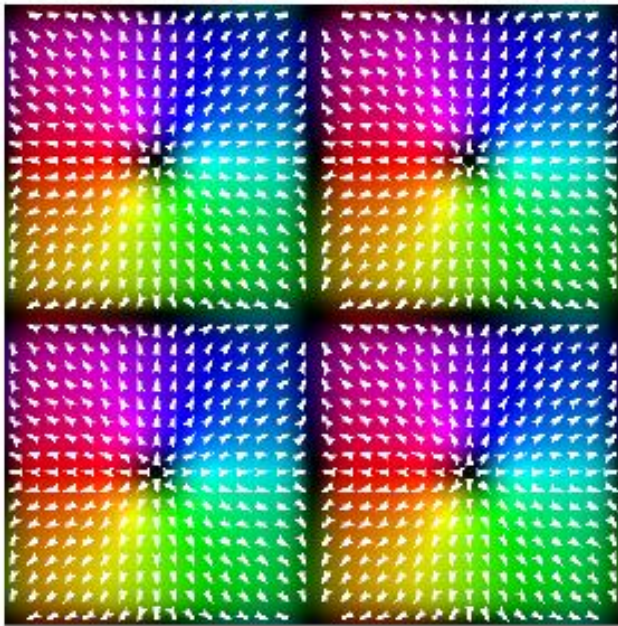
Topological charge,  $N_{sk} = \int_{u.c.} d^2r \chi(r) = -1_{30}$

# Square Skyrmion crystal

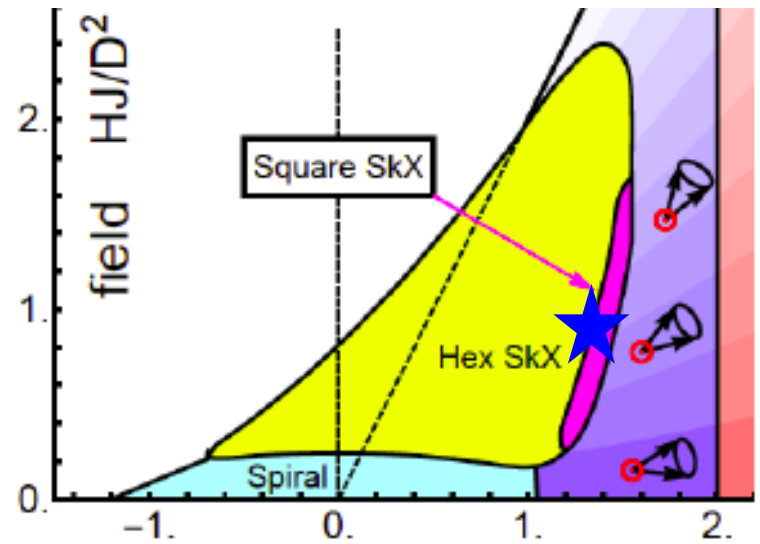
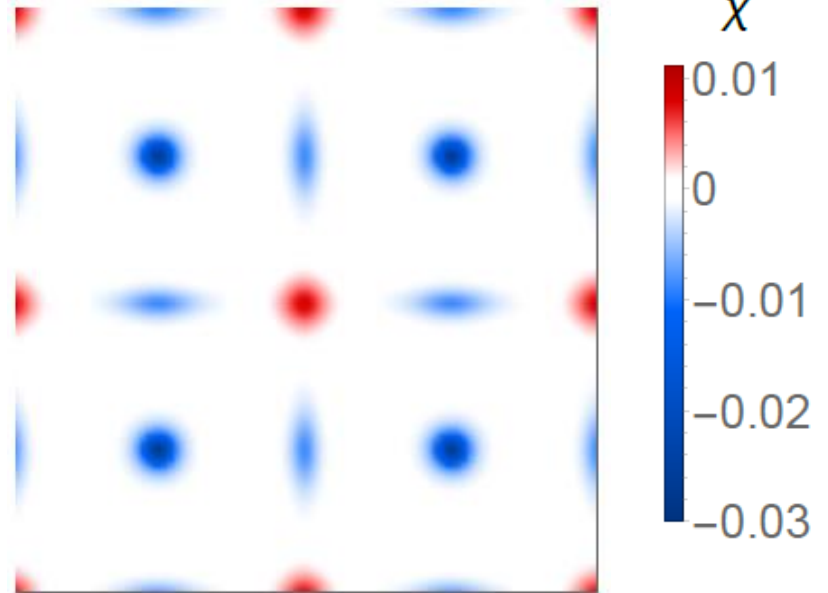
Topological charge density

$$\chi(\mathbf{r}) = \frac{1}{4\pi} \hat{\mathbf{m}} \cdot (\partial_x \hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}})$$

$AJ/D^2 = 1.4$



Topological charge  
Density  $\chi$

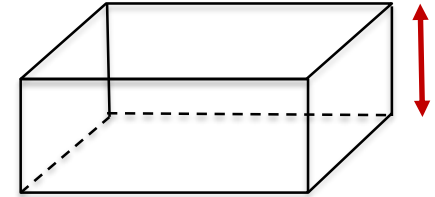


Topological charge,  $N_{sk} = \int_{u.c.} d^2r \chi(r) = -1$ <sub>31</sub>

## Systems with both Dresselhaus and Rashba DM terms

$$D_D = D \cos \beta$$

$$D_R = D \sin \beta$$



**3D systems:** Bulk or film with thickness  $\gg (J/D)a$   
→ periodic boundary condition in  $x, y, z$

e.g. Thick 3D chiral magnets under strain gradient

$$\mathcal{F}[\mathbf{m}(\mathbf{r})] = \frac{J}{2} |\nabla \mathbf{m}|^2$$

$$+ D \cos \beta \mathbf{m} \cdot (\nabla \times \mathbf{m})$$

$$+ D \sin \beta \mathbf{m} \cdot ((\hat{\mathbf{z}} \times \nabla) \times \mathbf{m})$$

$$+ A m_z^2$$

$$- H m_z$$

Isotropic FM exchange

DM -- Dresselhaus

DM -- Rashba

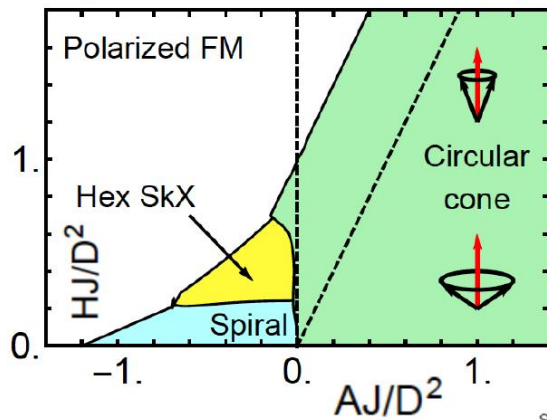
Anisotropy

Magnetic field

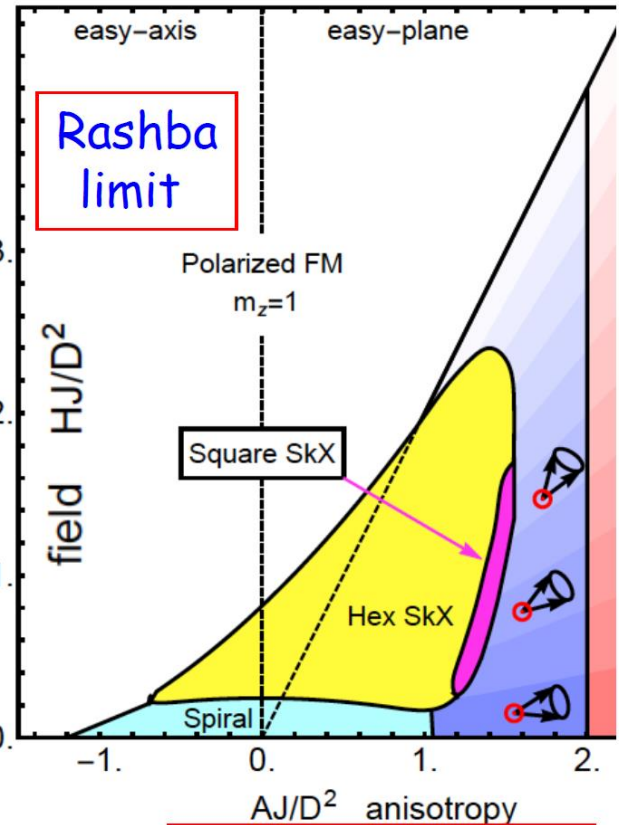
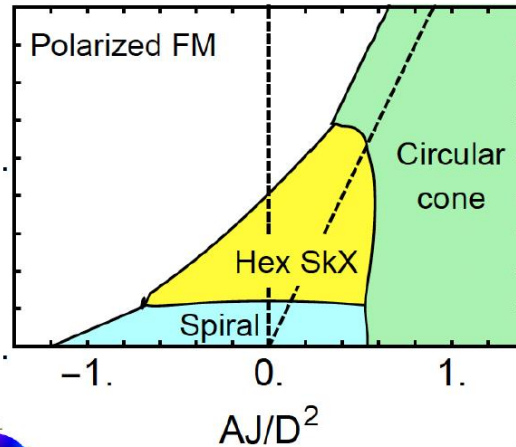


# Evolution from Dresselhaus $\rightarrow$ Rashba

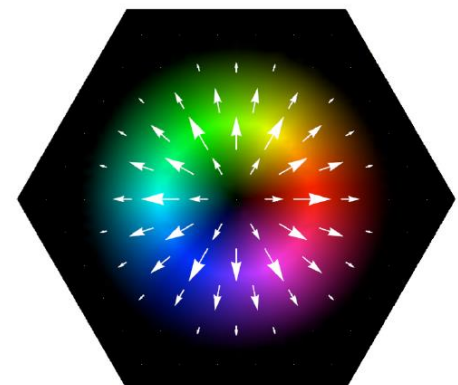
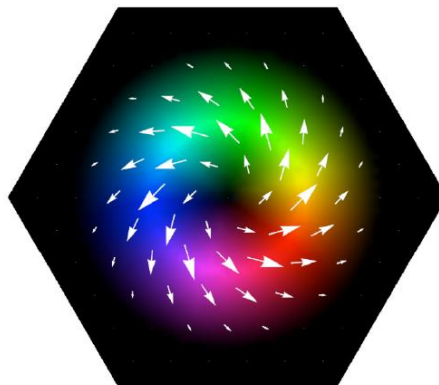
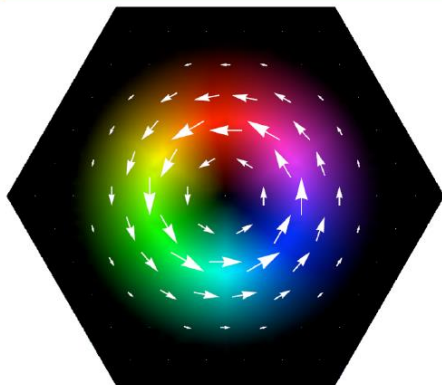
Dresselhaus limit



$$D_R = D_D$$



Vortex-like / Bloch



Hedgehog / Neel

# Why care about skyrmions?

→ Metallic chiral magnets

## “Emergent electromagnetism”

- Topological Hall effect
- Spin-transfer torque

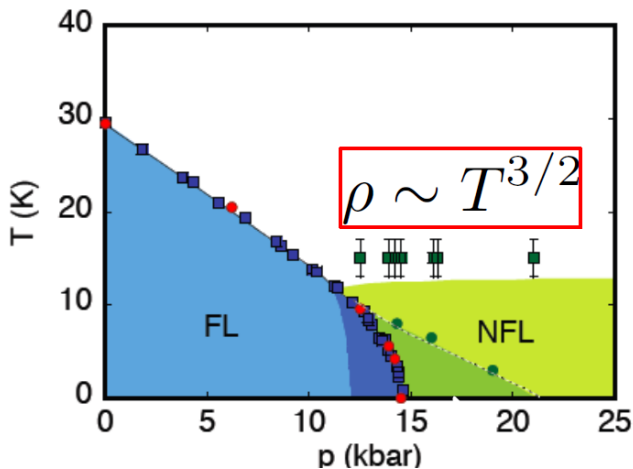
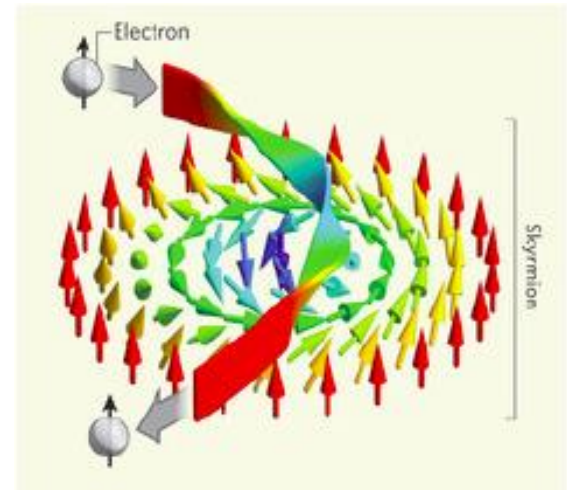
Manipulation of skyrmion motion by ultrasmall current.

- Low pinning compared to domain walls.
- Non-Fermi liquid phase under pressure

- Real-space Berry phase  
→ Emergent magnetic field

$$B_{eff}^z \propto \mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m})$$

~100 T for 10 nm skyrmion



## MnSi

Ritz et al,  
Nature  
497, 231 (2013)

Review: Nagaosa & Tokura,  
Nature Nano (2013)

# Conclusions

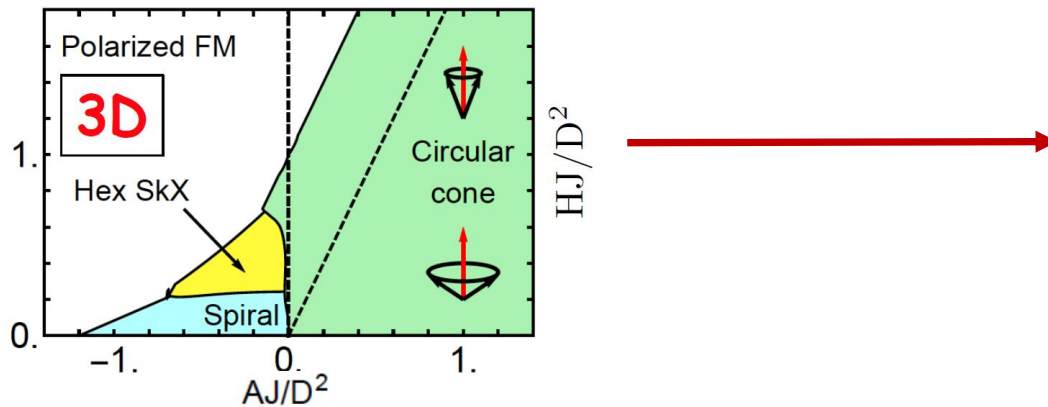
Enhanced stability of skyrmion phases →

3D → 2D

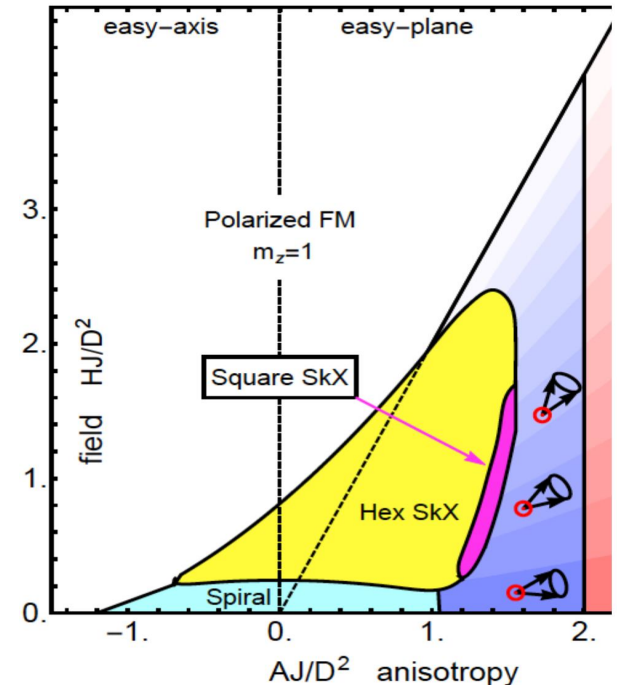
Broken bulk inversion → broken surface inversion/mirror

“Dresselhaus” SOC → Rashba SOC

- Role of easy-plane anisotropy



- Much richer skyrmion phase diagram  
e.g. Transition between SkX phases



Magnetic oxide interfaces

- Broken surface inversion
- Large Rashba SOC

Thank you!