Can a random harmonic chain thermalize strongly localized particles in one dimension?

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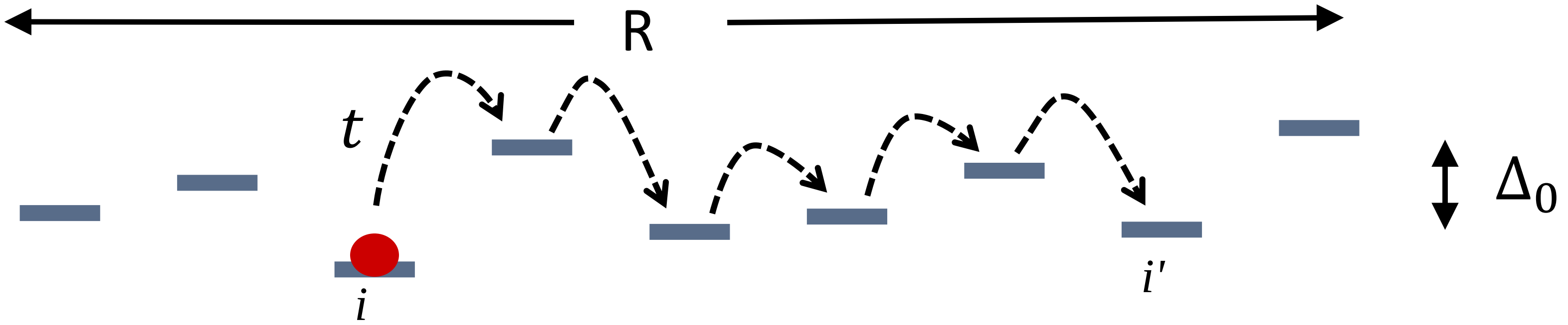
ultracold quantum matter

Anderson localization

- Single-particle localization

$$\mathcal{H} = -t \sum_{<ij>} (c_i^\dagger c_j + h.c.) - \sum_i V_i n_i$$



- Is there a possible resonance within a distance *R* of site *i* ?

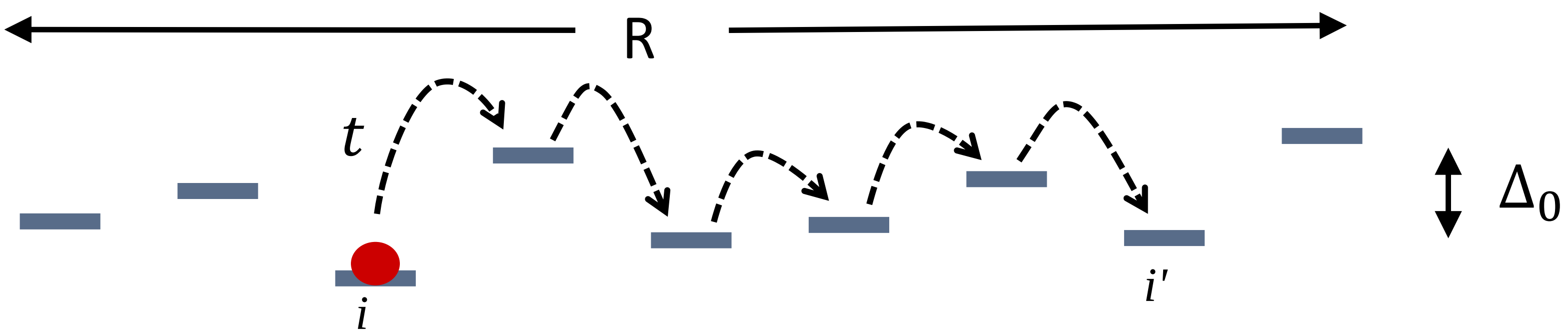
Site nearest in energy within this range, $\Delta_R \sim \Delta_0 / R^d$

Matrix element for hopping to this range: $J_R \sim t \left(\frac{td}{\Delta_0} \right)^R$

→ Resonance condition $J_R > \Delta_R$ satisfied only if $td > \Delta_0$

Anderson localization

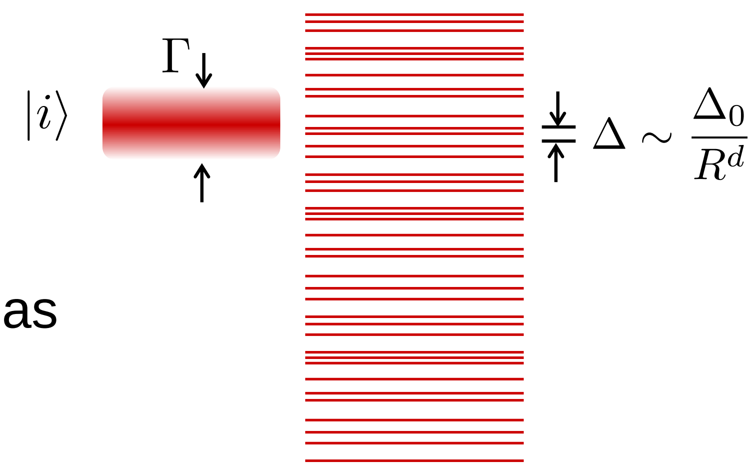
$$\mathcal{H} = -t \sum_{<ij>} \left(c_i^\dagger c_j + h.c.\right) - \sum_i V_i n_i$$



- Can view delocalization as a decay of state $|i\rangle$ into a continuum

  - Fermi golden-rule rate for decay

$$\Gamma_R \sim J_R^2 \Delta_R^{-1}$$



Condition for the surrounding states to serve as an effective bath:

$$\frac{\Gamma_R}{\Delta_R} = \left(\frac{J_R}{\Delta_R}\right)^2 > 1$$

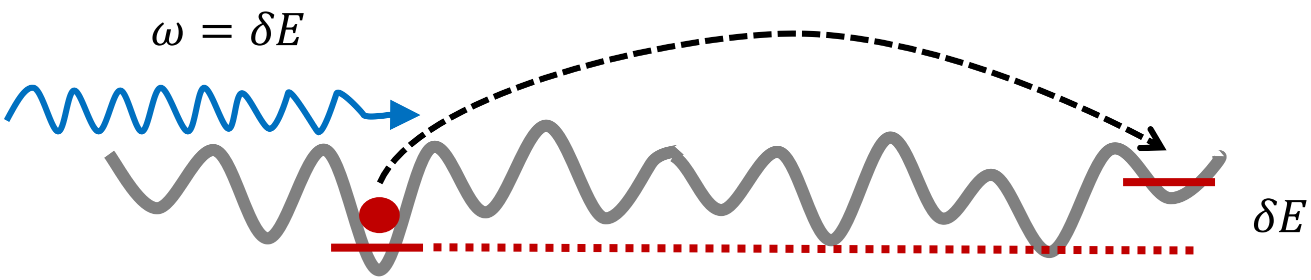
But, Anderson insulators in solids are not insulators!

- Coupling to delocalized phonons → Variable-range hopping

Conductivity in Anderson "insulators" (T>0)

Phonon assisted hopping (Mott):

$$\sigma = \sigma_0 e^{-\left(\frac{T_0}{T}\right)^{\frac{1}{d+1}}}$$



Closed system with interactions (no phonon bath) :

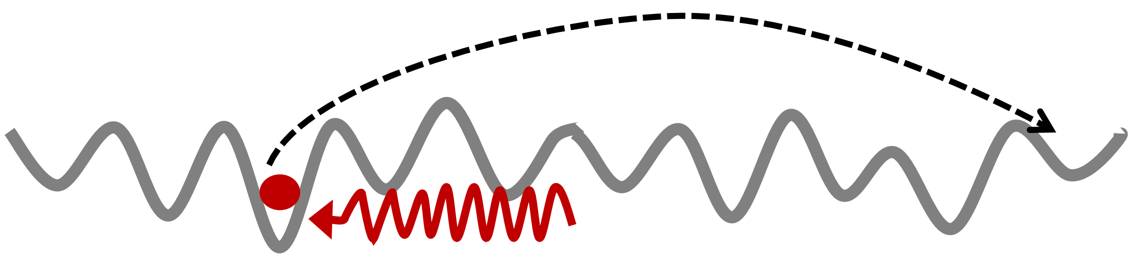
Question: Can collective excitations, e.g. particle-hole, in interacting system play the role of phonons?

Answer: No! for sufficiently strong disorder collective excitations are also localized

→ discrete local spectrum → fail to serve as a bath

→ Many-body localization (MBL)

Basko, Aleiner, Altshuler (2005)



Question

- Can phonons always delocalize electrons?

- Revisit Electron phonon problem with configurational disorder in 1d.

  All phonons are localized except at $\omega = 0$.

  Can we have Variable-range hopping in this case?

Relevant for fermions coupled to the excitations of a condensate with 1d disorder.

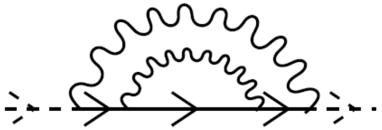
Outline

- Variable-range hopping.

- Why basic (perturbative) variable-range hopping process fails in 1d with configurational disorder?

- Refined analysis.

➢ High order perturbation theory in el-ph coupling.



➢ Non-perturbative: Polaron calculation.

  - Delocalization via arbitrary high-order phonon process as $T \to 0$.

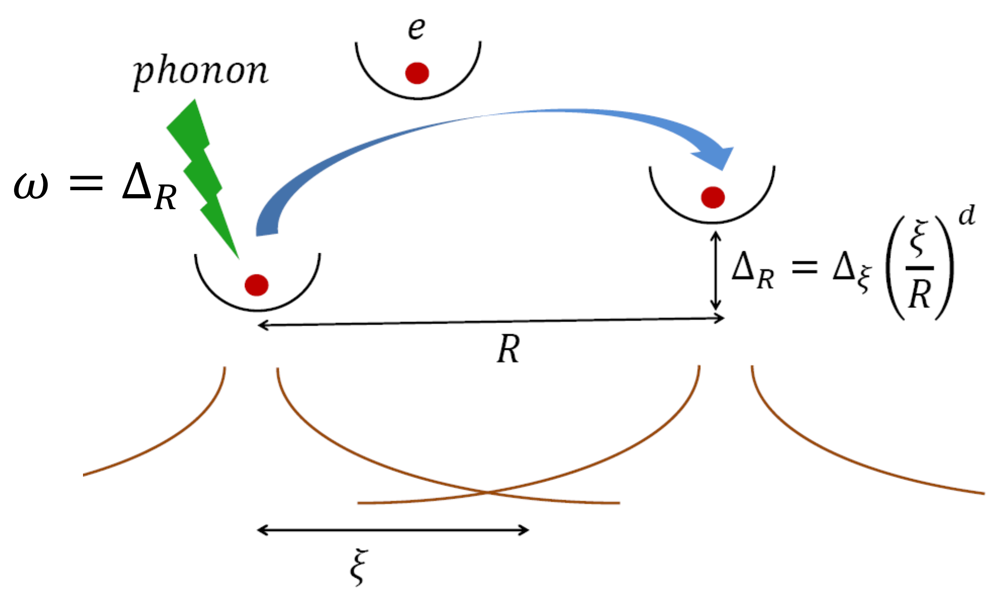
  ➢ Modified "variable-range hopping rate".

Phonon assisted hopping in disordered electronic system

- Phonon-assisted hopping rate of localized electrons

$$\frac{1}{\tau} \sim \sum_{R} g_R e^{-\left(\frac{R}{\xi}+\frac{\Delta_R}{T}\right)}$$

Fermi golden rule (FGR)
for one-phonon absorption



- Saddle point approximation

→ (Mott) Variable-range hopping (VRH) rate,

$$\frac{1}{\tau_M} \simeq g_M\, e^{-A_d\left(\frac{\Delta_\xi}{T}\right)^{\frac{1}{d+1}}}$$

$g_M \propto$ electron-phonon coupling.
d dimension

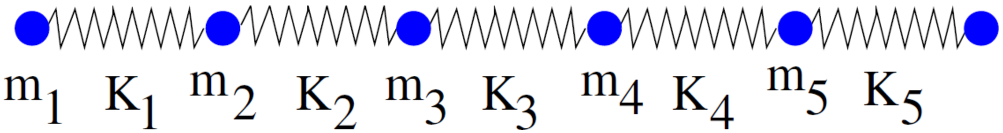
- Temperature-dependent (variable) hopping range → $R_M \simeq \xi\left(\frac{\Delta_\xi}{T}\right)^{\frac{1}{d+1}}$

Electron draws an `optimal' energy $\Delta_M \simeq \Delta_\xi \left(\frac{T}{\Delta_\xi}\right)^{\frac{d}{d+1}}$ from phonon bath.

← Dimension $d \geq 3$, all low-frequency modes are extended

Phonon localization in 1D random chain

Random harmonic chain
Disorder in masses and springs →



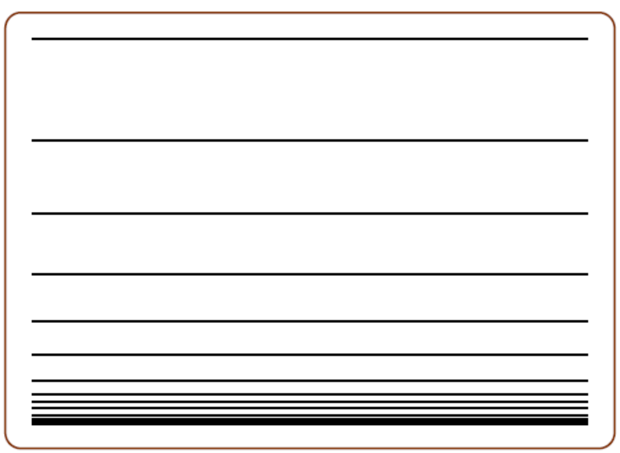
- Phonons are localized at all non-zero frequency in 1D
  → Phonon localization length

$$\ell_\omega \simeq \ell_0 \left(\frac{\omega_0}{\omega}\right)^\alpha$$

$2 \geq \alpha > 1$ (Weak → Strong disorder)

S. John et al. (1983), V. Gurarie et al. (2008)

- Low-energy phonon DOS $D(\omega) \simeq 1/c$, constant



'One-phonon' bath has discrete spectra

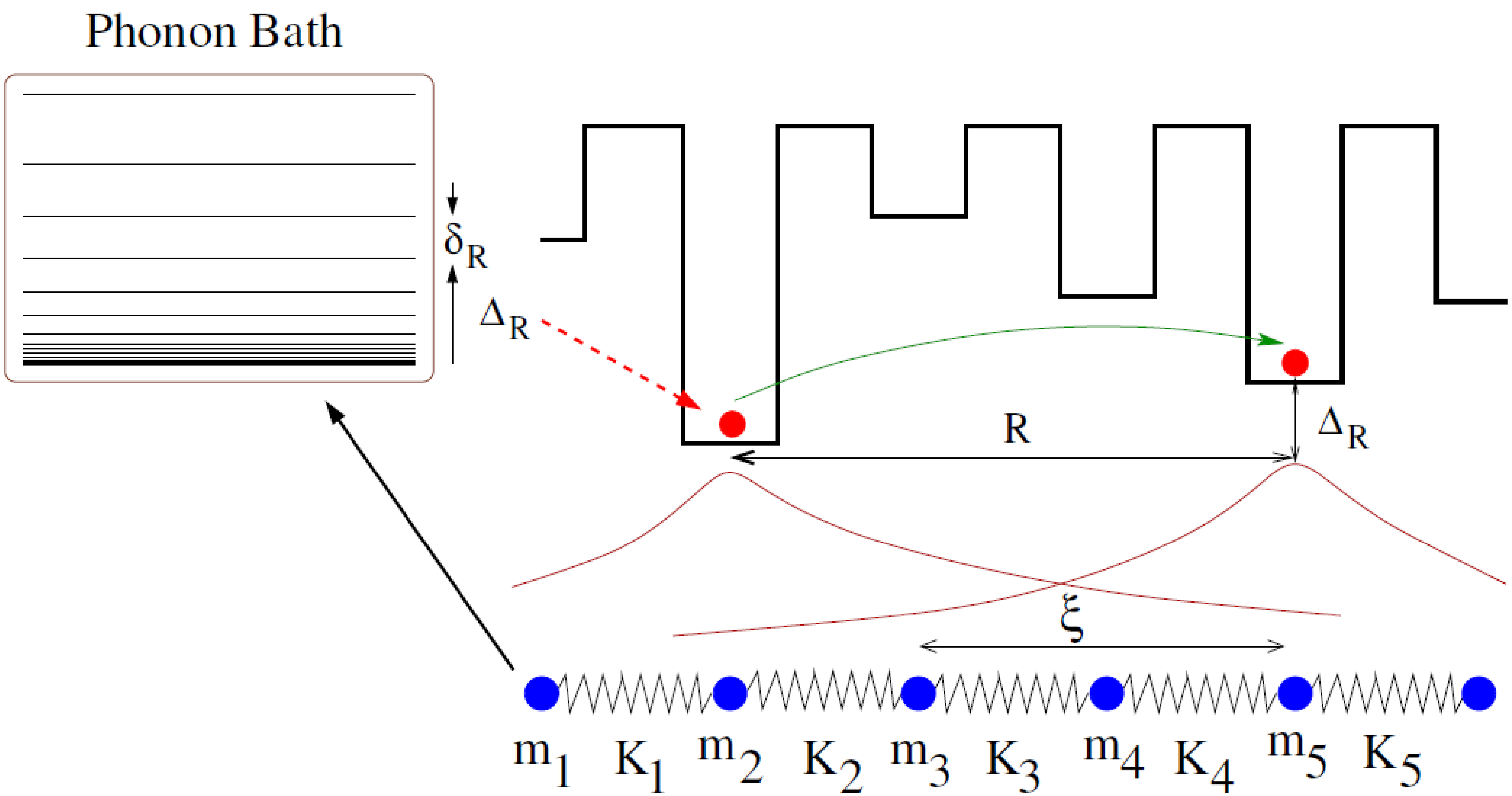
Level spacing →

$$\delta_\omega \simeq 1/\ell_\omega D(\omega) \sim \left(\frac{\omega}{\omega_0}\right)^\alpha$$

Questions

- How does phonon localization in a random harmonic solid affect VRH transport ?
- Is there a possibility of many-body localization (MBL) of coupled electron-phonon system?

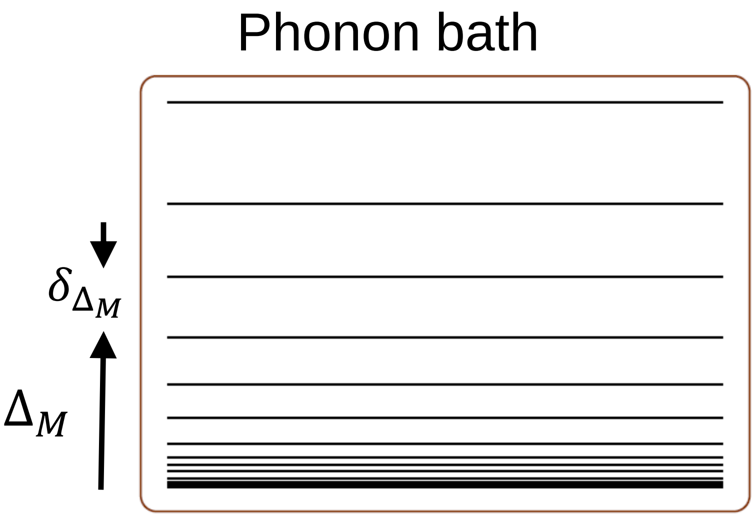
Localized particles coupled to random harmonic chain



Absence of variable-range hopping in 1D random harmonic chain?

- Fermi golden-rule rate is non-zero only if

$$\frac{1}{\tau_M} \gg \delta_{\Delta_M}$$



δ_{Δ_M} , phonon level spacing at the

absorbed energy $\Delta_M \simeq \Delta_\xi \left(\frac{T}{\Delta_\xi}\right)^{\frac{1}{2}}$

$$\rightarrow \frac{1}{\tau_M} \approx g_M \, e^{-2\left(\frac{\Delta_\xi}{T}\right)^{\frac{1}{2}}} > \tilde{\omega}_M \left(\frac{T}{\Delta_\xi}\right)^{\frac{\alpha}{2}} \;??$$

- The level spacing is larger at low temperature ($T \rightarrow 0$) and could be made larger over the entire VRH temperature regime
  - Fermi golden-rule rate is zero

Many-body localization of electron-phonon system ?

Wait! What happens to higher-order phonon processes?

Level spacing for n-th order phonon process

$$\delta_\omega^{(n)} \approx \widetilde{\omega}_n \left(\frac{\omega}{\omega_0}\right)^{\alpha^n}$$

→ Level spacing relevant for higher-order phonon processes decreases very rapidly with number of phonons absorbed (emitted).

Need to compare higher-order rates with relevant level spacings for many-phonon process.

Model

$$\mathcal{H} = \mathcal{H}_{el} + \mathcal{H}_{ph} + \mathcal{H}_{el-ph}$$

- $\mathcal{H}_{el} = \sum_l \epsilon_l c_l^\dagger c_l$

Localized electronic state: $\epsilon_l,\ \psi_l(x) \sim e^{-\frac{|x-x_l|}{2\xi}}$

- $\mathcal{H}_{ph} = \sum_\mu \omega_\mu a_\mu^\dagger a_\mu$

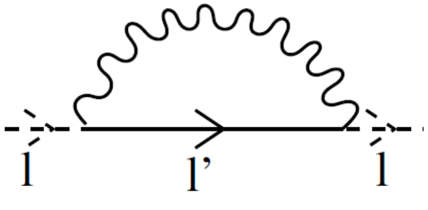
Localized phonon mode: $\omega_\mu,\ \Phi_\mu(x) \sim e^{-\frac{|x-x_\mu|}{2\ell_\mu}}$

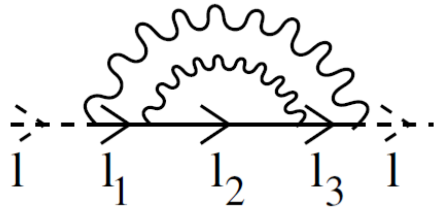
- Electron-phonon coupling

$$\mathcal{H}_{el-ph} = \sum_{x,\mu} g_\mu(x) n(x)\ (a_\mu^\dagger + a_\mu)$$

$g_\mu(x) \approx \frac{g}{\lambda_\mu \ell_\mu^{\frac{1}{2}}} \left(\frac{\omega_0}{\omega}\right)^{\frac{1}{2}} e^{-|x-x_\mu|/2\ell_\omega}$, $\lambda_\omega \simeq c/\omega_\mu$, phonon wavelength

Hopping rate: Weak el-ph coupling





Rate,

$$1/\tau_l \sim -Im\Sigma_{ll}(\epsilon_l + i0^+)$$

Saddle point approximations →

- First order (one-phonon) rate (usual VRH rate)

$$\frac{1}{\tau_M^{(1)}} \approx g_M^{(1)}\, \tilde{T}^{\frac{1}{2}}\, e^{-2/\sqrt{\tilde{T}}} > \delta_{\Delta_M}^{(1)} \approx \tilde{\omega}_1\ \ \tilde{\omega}_0^{-\alpha}\ \tilde{T}^{\frac{\alpha}{2}}$$

$$\tilde{T} = \frac{T}{\Delta_\xi} \ll 1, \tilde{\omega}_0 = \frac{\omega_0}{\Delta_\xi} > 1$$

- Second order (two-phonon) rate, optimal energy $\Delta_M \simeq \Delta_\xi \left(\frac{T}{\Delta_\xi}\right)^{\frac{1}{2}}$

$$\frac{1}{\tau_M^{(2)}} \approx g_M^{(2)}\, \tilde{T}^{\frac{3}{2}}\, e^{-2/\sqrt{\tilde{T}}} > \delta_{\Delta_M}^{(2)} \approx \tilde{\omega}_2\ \ \tilde{\omega}_0^{-\alpha^2}\ \tilde{T}^{\frac{\alpha^2}{2}}$$

Rate is also zero at second order

May need to go to very high-order phonon process → Strong-coupling approach

Small-polaron hopping

Small–polaron in random harmonic chain

- Polaron transformation

$$\bar{\mathcal{H}} = e^{S}\mathcal{H}e^{-S}$$

- Polaron operator

$$\bar{c}(x) = e^{S}\, c(x)e^{-S} = c(x)\chi(x)\;,$$

$$\chi(x) = e^{-\sum_{\mu}\frac{g_{\mu}(x)}{\omega_{\mu}}(a_{\mu}^{\dagger}+a_{\mu})}$$

- Polaron hopping rate, Fermi golden rule

$$\frac{1}{\tau_l} = \frac{2\pi}{\hbar}\sum_{f\neq i}\rho_{ph}\left|\langle f\middle|\hat{T}\middle|i\rangle\right|^{2}\delta(E_f - E_i)$$

Contains arbitrary high-order phonon process

$$\hat{T} = t\sum_{x,\delta}\chi^{\dagger}(x)\chi(x+\delta)c^{\dagger}(x)c(x+\delta)$$

Polaron hopping rate

- Fermi golden-rule rate

$$\frac{1}{\tau_l} = \frac{2\pi}{\hbar}\sum_{f\neq i}\rho_{ph}\left|\langle f\middle|\hat{T}\middle|i\rangle\right|^2\delta(E_f - E_i) \sim \tilde{t}^2\sum_R e^{-\frac{R}{\xi}}S(\Delta_R)$$

- $S(\omega) = \int_{-\infty}^{\infty} dt\, e^{-i\omega t}\,\langle\chi^\dagger(x,t)\chi(x+\delta,t)\chi^\dagger(x+\delta,0)\chi(x,0)\rangle$

➢ $S(\omega)$ constitutes the effective bath DOS.

Check the validity of Fermi golden rule rate.
→ Compare $1/\tau_l$ with the level spacing of $S(\omega)$.

Rate for *n*-phonon process

$$\frac{1}{\tau_l} \sim \tilde{t}^2 \sum_R e^{-\frac{R}{\xi}} S(\Delta_R) \ \sim \tilde{t}^2 \sum_R e^{-\frac{R}{\xi}} \sum_{n=1}^{\infty} \frac{\bar{g}^n}{n!} I_n(\Delta_R)$$

Expand the golden-rule rate in order of number (n) of phonons.

- $I_n(\Delta_R) \sim \int \prod_{i=1}^{n} d\omega_i \, \omega_i \, \mathrm{csch}\left(\frac{\omega_i}{2T}\right)$
  $\times \left[\delta(\Delta_R - (\omega_1 + \omega_2 + .. + \omega_n)\right) + \delta(\Delta_R + \omega_1 - (\omega_2 + .. + \omega_n)) + \cdots]$

← Contribution from *n*-phonon absorption (emission) processes.

- *n*-phonon assisted hopping rate $(n \gg 1)$

$$\frac{1}{\tau_M^{(n)}} \sim g_M^{(n)} \, \widetilde{\omega}_0^{-n\alpha} \, \tilde{T}^{\frac{n\alpha}{2}} e^{-\frac{2}{\sqrt{\tilde{T}}}}$$

Modified variable-range hopping rate

→ Compare the rate with level spacing of n-phonon bath

$$\frac{1}{\tau_M^{(n)}} \sim g_M^{(n)}\, \tilde{\omega}_0^{-n\alpha}\, \tilde{T}^{\frac{n\alpha}{2}}\, e^{-\frac{2}{\sqrt{\tilde{T}}}} > \delta_{\Delta_M}^{(n)} \approx \tilde{\omega}_n\;\, \tilde{\omega}_0^{-\alpha^n}\, \tilde{T}^{\frac{\alpha^n}{2}}$$

The level spacing could be made smaller than the rate for

$$n \geq n_c(T) \simeq \frac{\ln \tilde{T}^{-\frac{1}{2}}}{\ln \alpha}$$

- Order of the process diverges as $T \to 0$.

Modified variable-range hopping rate

$$\frac{1}{\tau_M} \sim \left(\frac{T}{\Delta_\xi}\right)^{\frac{\alpha}{4\ \ln \alpha}\ln\left(\frac{\Delta_\xi}{T}\right)} e^{-2\left(\frac{\Delta_\xi}{T}\right)^{\frac{1}{2}}}$$

Singular and highly suppressed pre-exponential factor

Higher dimension

Two-dimension:

- For weak disorder, the phonon localization length, $\ell_\omega \sim \ell_0 e^{\left(\frac{\omega_0}{\omega}\right)^2}$.

S. John et al. (1983)

→ one-phonon level spacing decreases very rapidly with energy.

→ usual VRH rate through one-phonon process is non-zero.

- ➢ What happens for strong disorder in 2d?

Three-dimension:

All low-frequency modes are extended.

→ usual VRH rate.

Conclusions

- The absence of usual VRH transport via one- and a few-phonon processes due to discrete nature of the phonon bath for localized particles coupled to 1d random harmonic chain.

- Very high-order process involving large number of phonons do eventually thermalize Anderson insulator.
  → Order of the process diverges as $T \to 0$.

- Very slow hopping rate due to highly suppressed pre-exponential factor of VRH rate.